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Kitāb Taḥdīd al-Amākin

The Determination of the Coordinates of Positions for the Correction of Distances between Cities

a translation from the Arabic

of

AL-BĪRŪNĪ'S

Kitāb Taḥdīd Nihāyāt al-Amākin Litaṣḥīḥ Masāfāt al-Masākin

by

JAMIL ALI

PREFACE

The Publications Committee for the 1966-67 Centennial Celebration of the American University of Beirut have commissioned me to translate, for the first time, the **Kitāb taḥdīd nihāyāt al-amākin li-taṣḥīḥ masāfāt al-masākin,** by Abu al- Raiḥān Muḥammad bin Aḥmad al-Bīrūnī, perhaps the greatest scientist of his day.

Al-Bīrūnī was born on the 4th of September 973, in the territory of Khwārizm, the modern Khiva, and died, probably at Ghazna, sometime after 1050.

The author spent the early part of his life in Khwārizm, as counsellor at the court of the ruling princes of the House of Ma'mūn, who were vassals of the kings of Central Asia of the House of Sāmān. But civil war broke out in 995, despair and danger loomed over him, and he had to seek safety outside his homeland.

Around 996 he arrived at Rayy, where he spent about two years in unfortunate circumstances. But in 999 he was welcomed in Jurjān (Gurgan) at the court of prince Qābūs bin Washmgīr Shams al-Maᶜāli, who ruled over Jurjān and the adjoining countries. To this prince he dedicated, in 1000 A.D., his major work on chronology: **Al-āthār al-bāqiyaᶜan al-qurūn al-khāliya.**

During the years 1010-1017, he lived again in Khwārizm at the court of Abū al-ᶜAbbās al-Ma'mūn, as his friend and counsellor. But a rebellion broke out in 1017, Abū al-ᶜAbbās was murdered, and the country was conquered by Sultan Maḥmūd of Ghazna, who annexed it to his empire. On returning to Afghanistan, in 1018, he carried off al-Bīrūnī and other scholars as hostages to Ghazna.

Soon after his arrival in Ghazna, the author began his composition of the Taḥdīd. He says at the beginning of the third chapter that at the time of writing, on the 1st of October 1018, he was in Jayfūr, a

village near Kābul. We also learn from the colophon that the Taḥdīd was completed at Ghazna, on the 28th of August 1025.

As counsellor and court astrologer, al-Bīrūnī accompanied Sultan Maḥmūd on many of his expeditions into India, which provided the author with the opportunity of studying its geography and history, and the language, literature, philosophy, manners, and customs of the Hindus.

In 1030, the author was back in Ghazna, and worked on his astronomical magnum opus: **al-Qānūn al-Masᶜūdī,** which he dedicated to Masᶜūd, a son of Sultan Maḥmūd, on his assumption of power in 1031.

In the prime of his intellectual life, about 1032, he composed his famous memoir on India, **Tārikh al-Hind.** For his critical faculty, tolerance, objectivism, and intellectual courage, he is regarded one of the greatest Indologists.

Besides these major studies, he wrote smaller ones on a variety of subjects: **Kitāb al-Tāfhīm,** an exhaustive compendium of astrological lore; **Kitāb al-Jawāhir,** an opuscule on mineralogy and the study of gems; **Kitāb al-Ṣaidana,** a kind of materia medica of his days. The titles of 148 works of al-Bīrūnī are known, and about 32 of these are extant to this day.

His work exhibits profound erudition of encyclopedic scope, originality, and trenchant humour; and it demonstrates also, in addition to his Khwārizmian vernacular, a good literary command of Arabic, Persian, Sanskrit, and Syriac.

The present translation of the Taḥdīd is based on Dr. P. Boljakoff's edition of the Istanbul manuscript, Sultan Fātiḥ No. 3386, which was published, in 1962, by the Cultural Department of the Arab League, Cairo. We are sorry to state that we were not able to procure a copy of the edition of the manuscript by Muḥammad bin Tāwīt al-Ṭanjī, which was published in Ankara in 1962.

The text deals with geographical-mathematical subjects: the determination of latitude, the obliquity of the ecliptic, the measurement of time, lunar eclipses, longitudinal differences, and methods for the determination of the **qibla,** the direction of prayer for muslims.

At the end of the first chapter, the author declares that his intention is to determine especially the position of Ghazna on the surface of the earth. He had in mind the construction of astronomical

tables, based on Ghazna, in order to give this city and the reigning Ghaznavids a scientific standing corresponding to its political importance.

Towards the end of the sixth chapter, the author reports the measurement of a degree of the meridian by al-Ma'mūn's astronomers at Sinjār, near Mosul. He also gives a method of his own invention for measuring the radius of the earth, which he applied successfully at the fort of Nandana during his stay in India.

I am sorry to report that some of the calculation in the text is inaccurate. For example, the ciphering on page 257 of the Cairo edition is erroneous. According to the editor of this edition, the Istanbul manuscript is not an autograph written by al-Bīrūnī, because of the frequent incidence of errors in the lettering of the diagrams, and the numerous notes added in the margins or inserted between the lines. So, if the ciphering was not performed by an assistant, perhaps the inaccuracies sprang from the necessity of performing, by the great scholar himself, the stupendous mass of calculations in his work.

I have not wished to burden the text with footnotes, and my parenthetical interpolations are not numerous. The numbers at the centres of spaces between the lines give the serial order of pages in the Cairo edition; those in the left-hand margin of a specified page of that edition give the serial order of lines, in multiples of three. Also, a number **n** in the margin indicates that the translation of the line of serial order **n** starts somewhere along the line opposite that number. Similarly, (MS**n**) indicates the start of a page of the Istanbul manuscript of serial order **n.**

It is my pleasure to announce that my colleague, Professor E.S. Kennedy, is writing a commentary on the Taḥdīd which will be published in due time. I have constantly referred to him for advice, and he has kindly read and corrected the whole of the present translation. For the time and thought he has so generously given to me, I am indeed heavily indebted.

I am also deeply thankful to my colleague, Professor P. Yff, who has corrected many errors, and made many valuable suggestions and criticisms. For his critical reading of the proofs of the entire book, I am sincerely grateful.

I desire to express my gratitude to the General Editor of the Centennial Publications, Dr. F. Ṣarrūf, for his skilled editorial judgment,

to Mrs. S. Tūqān, Dr. Ṣarrūf's assistant, for reading the proofs of the whole book, to Mrs. K. Shomar, secretary of the Mathematics Department, for typing the final script, to Mr. Victor Ṣāyigh of the School of Engineering for drawing the diagrams, and to the officials of the National Lebanese Printing Press for their unfailing courtesy and good work.

Finally, the responsibility for all errors in the translation is mine, and I ask the reader to pardon my error and correct it.

Jamil Ali

February 1967

TRANSLITERATIONS
ADOPTED IN LETTERING THE DIAGRAMS

أ	A	م	M
ب	B	ن	N
ج	G	س	S
د	D	ع	O
ه	E	ف	F
و	W	ص	C
ز	Z	ق	Q
ح	H	ر	R
ط	T	ش	X
ى	Y	ت	Θ
ك	K	ث	Σ
ل	L	خ	Γ

CONTENTS

22 (MS 1)

IN THE NAME OF GOD,
MOST GRACIOUS, MOST MERCIFUL

In this book, **Taḥdīd nihāyāt al-amākin litaṣḥīḥ masāfāt al-masākin,** (Determination of coordinates of positions for the correction of distances between cities), Abū Raiḥān Muḥammad bin Aḥmad al-Bīrūnī says :

Minds have a pressing need for expanding their domains of activities, and souls can not be satisfied without spiritual contemplation. Hence it is my ambition to present what passes through my mind concerning the creation of an art, or the perfection of the projected shadow of knowledge; so that on beholding it the mind shall see the shadow most beautifully clothed, and shall find in it the satisfaction which is inspired by its virtues of permanence for all time. Whoever succeeds in such an achievement is a man of admirable character, and has the virtues of excellence (both) relatively and absolutely.

I feel I can almost trust the findings of the astrologers concerning the cycles and rules that govern the motions of hundreds and thousands of the stars, and that events all over the world are governed by those same rules; when I behold how the people of our time, in different parts of the world, reflect such forms of ignorance, and how proud of it they are; how antagonistic to people of excellent virtues they are; how they plot against a learned scholar and inflict on him all sorts of harm and persecution.

Also, people have agreed - though a whole nation does not agree to go astray - to adopt the worst code of morals, and to approve of indiscriminate greed, which is most (MS 2) harmful to a community as a whole. You do not see among them anything except hands outstretched for begging, with no sense of shame or pride to curb them from doing mean deeds. In fact they compete in this shameful greed; they look for opportunities that whet their desires for it, and this has led them to discard the sciences and to persecute the custodians of learning.

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The extremist among them would stamp the sciences as
atheistic, and would proclaim that they lead people astray in
order to make ignoramuses, like him, hate the sciences. For this
3 will help him to conceal his ignorance, and to open the door for
the complete destruction both of sciences and scientists.
The rude and stubborn critic among them, who calls himself
impartial, would listen to scientific discourses, but his persistent
stubbornness ultimately reveals the meanness of his forebears.
He would come forth with what he considers to be great wisdom,
and say: «What is the benefit of these sciences?» He does not
6 know the virtue which distinguishes mankind from all sorts of
the animal kind: it is knowledge, in general, which is pursued
solely by man, and which is pursued for the sake of knowledge
itself, because its acquisition is truly delightful, and is unlike
9 the pleasures desirable from other pursuits. For the good cannot
be brought forth, and evil cannot be avoided, except by knowl-
edge. What benefit then is more vivid? What use is more
abundant? In spiritual as well as in wordly affairs, we cannot
be sure, without knowledge, that what we seek and bring forth
is the good, and that that which we avoid is evil.
If worldly possession is the benefit mentioned heretofore,
12 then one can attain such a benefit in three ways: (a) Safe occup-
ations which do not prosper without some knowledge and are
morally sound vocations: such as the governorship of a district,
business and commerce in general, (MS 3) and real estate broker-
age. (b) Dangerous occupations: such as alchemy, falsification
of documents, theft of jewelry, cheating in selling, theft of
15 money, and robbing by choking. (c) The third way is pursued
by him whose excessive greed has extinguished the lights in his
heart

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and mind: such as selling wine, the prostitution of males and
females, and the procurement of kith and kin for illicit sexual
intercourse with strangers. Such a villain would not avoid such
despicable pursuits; he would probably find all sorts of excuses
3 for their approbation, and for him these are delightful activities,

because they would provide him with the immense material
benefits which he sought.
I do not think a benefit seeker is concerned about the good
6 for his soul in the hereafter. Suppose he is concerned, then it is
well known that primitive worship does not do his soul any
good, because it is not based on intelligent knowledge which
discriminates good from evil. Such primitive worship abounds
in different parts of the world, and is practised by different
nations. Hence it is a cause for dissent, because truth can not be
reached by worships based on different dogmas. If a worshipper
9 is a truth seeker, then he is eventually led to an investigation of
the old and new conditions of the world, and if he ignores that
investigation, he can not pursue truth without reading intelli-
gently about the rules of order in the universe and its parts, and
without investigating the validity of those rules. This investig-
12 ation will acquaint him with the Maker and His deserving
qualities; and the knowledge of these qualities is the **sine qua
non** for the recognition of a revealed prophecy, (MS 4) because
their identification is essential to discriminate between a proper
and a pseudo prophet. The pretenders to revelations are many,
and since they do not agree among themselves, it is certain
that some of them are false pretenders who would lead people
astray.
15 This point of view is that which God Almighty would
accept from His servants. He says, and what He says is luminous
truth: «And contemplate the (wonders of the) creation in the
heavens and the earth.» «Our Lord ! Not for naught hast Thou
created (all) this!» (Qur'ān, Sura 3:191). This noble verse contains

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the totality of what I have explained in detail, and until a man is
truly disciplined by its instructions he can not truly realize the
essence of wisdom in all forms of science and knowledge. There
are two alternatives: either he would take it as a story and a
tradition, or he would scientifically investigate and realize the
3 truth it contains. But surely there is a great difference between
an investigator of truth and a follower of tradition. God says:
«Are those equal, those who know and those who do not know?

It is those who are endowed with understanding that receive
admonition» (Qur'ān, Sura 39:9). A traditional follower of these
principles is as ignorant as a traditionalist who follows what is
6 derivable from them. God Almighty guides to the truth!
As to the sciences, man was naturally inclined to accept
them, because during his lifetime he could only fulfil certain
9 specific functions. The diversity of his needs, his insatiable
desires, and his lack of instruments for defending himself against
his many enemies, have all forced on him the inevitability of a
civilized life with his human kind, that is, to cooperate in the
division of labor, so that each (MS 5) may work for himself, as
well as for others. Everyone needed something that can be
12 divided into parts, and another which can be accumulated by
duplication. Hence labor was divided according to need. But as
labor and needs were disproportionate, and the times of labor
involved were unequal, people devised systems of prices and
exchanges which were based on the intrinsic worth of metals,
15 jewels, and things resembling them, which are pleasant to look
at, durable, and of rare occurrence. The exchanges and prices
were made on the just basis of labor involved, which thieves
and oppressors do not disregard, and which even waterfowls
like the **burak** (swans?) and the pelicans uphold. When these
18 birds fish in shallow water, they divide into two groups: one
group flaps its wings

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on the water and drives the fish towards the other group which
traps it, but does not devour it without sharing the catch with
the chasing group. They actually gather the fish in sacks at the
3 backs of their mouths, then discharge them and distribute them
equally. Glory be to God Almighty!
Further, I say that a civilized person is keen to possess
6 worldly valuables: «Heaped up hoards of gold and silver, horses
branded, and cattle and well-tilled land» (Qur'ān, Sura 3:14).
Hence the need arose for sciences which enable people to check
the amount and area of possessions, when transferred (MS 6)
from one owner to another, either by exchanges, or by inheritance.
9 The principles of these sciences are called mathematics and

formulae; they are embodied in the science of geometry, and
its benefit has just been mentioned.
Also, as man breathes the air which may carry various
12 kinds of infections, and as he lives on water and plants of various
qualities, and as he is subject to heavenly and tellurian cataclysms
which invade him from without, and storm upon him from
within, and as the repulse of some of these mishaps is possible,
and as there is an antidote to every evil (?), so man's experiments
and investigations have led him to build up the sciences for
15 medical and veterinary services. This organic science developed
as time went on, and mankind and most animals have benefited
by its development, but its achievements have remained insig-
nificant when they are compared with those of absolute
science.
As the wealthy do not live without musical entertainments,
18 and as those not wealthy are (even) keener on those entertain-
ments, and since ascetics are permitted to listen to music, and
as the influence of music is deeper when it is based on the rules
of harmony because the soul appreciates order and is (MS 7)
more responsive to poetry than prose because of poetry's rhyth-
mic structure, and in particular to poems set to the accompani-
ment of music,

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because of the similarity of structure between poetry and musical
harmony; so the mathematicians have investigated and discover-
ed the true principles of the science of music.
3 I say, further, that man's instinct for knowledge has cons-
tantly urged him to probe the secrets of the unknown, and to
explore in advance what his future conditions may be, so that
he can take the necessary precautions to ward off with fortitude
6 the dangers and mishaps that may beset him. Also, as the in-
fluence of the sun in the atmosphere changes in cycles which
revolve with the seasons, and as the influences of the moon on
the seas and the rains are cyclic, which revolve with her quarters
and the nychthemerons; so man has extended his experiments
and observations to the stars in the universe, other than the sun
9 and the moon, and built up the science of astronomy, with its

special methods of observation, without much trouble, or external complications.

As man has the gift of speech, and as he is an argumentative dialectician on worldly affairs, and those of the hereafter; so his speech has to conform to certain rules, because the statements he makes may be either false or true. Therefore (MS 8) he devised the science of logic, which is based on the compound syllogism in dialectic to discover the truth or error in a given premise or proposition, and to correct an erroneous inference.

I am surprised when I see one who hates logic and calls it by opprobrious names, when he fails to understand it. If such a person

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shakes off his laziness, puts in some effort, and reads some grammar, prosody, and logic, which form the principles of speech, he will know that speech itself is divided into prose and verse. In grammar, we find the rules for correct prose construction, and in prosody the rules for verse construction, but the rules of grammar are more general, because they apply to both prose and verse.

Now, speech in both divisions is expressed in statements which convey the sense intended by the speaker, and when statements are composed in a syllogism, a sense is either asserted or negated. Thus logic and its syllogisms give the rules for that composition and, generally speaking, it is like grammar. The three of them are excellent race horses, and any adverse criticism of one of them applies equally to the others.

The science of logic is attributed to Aristotle, and some of his theories and beliefs run contrary to the beliefs of Islam, because he was a theoretician and not a theologian. The Greeks (MS 9) and the Romans of his days were idolaters and star worshippers, so the extreme fanatics of our days call everyone whose name ends with (the letter) **sīn** a rejecter of Islam and an atheist. The **sīn** does not occur in the original Greek name; it appears in the transliteration into the Arabic, and is tacked on to the end to facilitate the pronunciation of the name, when

it is used in the nominative case. But ignoring an achievement and distorting it out of hatred for its author, and ignoring the truth in one case because its author had erred in another are both contrary to what God, be He exalted, says: «Those who listen to the Word and follow the best (meaning) in it; those are the ones whom God has guided» (Qur'ān, Sura 39:18). Yes, logic was written in words similar to the original Greek words, and in terms unfamiliar to the moderns, and because the subject itself is intricate, people have found it difficult to understand, and hence have avoided it.

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We see that the moderns use the methods of logic in their dialectic, dogmatic theology, and jurisprudence, but they use their common words and, therefore, they do not hate those methods. If you mention the words: **eisagoge, categorias, peri-harmonias,** and **anulutika,** the modern scholars would deplore them – «Looking at Thee with a look of one in swoon at the approach of death» (Qur'ān, Sura 47:20). They are right; it is the crime of the translators, (MS 10) for if they had used the equivalent Arabic terms: **al-madkhal, al-maqulāt, al-ʿibāra,** and **al-qiyās wal-burhān,** the scholar would not have avoided them, and would rapidly have adopted these terms.

What has been said is an exposition of the sciences; they were created by the urgent needs of man and developed accordingly. Their benefits are the satisfactions of those needs, for neither silver nor gold can satisfy them.

We now discuss Arabic rhetoric. If someone asks about its benefit, it is literary excellence itself, of which the Prophet, God bless him, has said: «Rhetoric is a form of magic». Rhetoric has also established the incomparable literary excellence of the Qur'ān, which is the source of Islam, and of belief in its revelation. Some people use rhetoric to impress people of influence, and thus attain high favor which enables them to realize their desires in worldly possessions and power, working gradually to the wazirate, which is second to the caliphate only. But Arabic rhetoric may lose its charm when it is applied to another language; it may make its author

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an object of ridicule, and thus may bring him ruin instead of material benefits which give security from want. This does not belittle the intrinsic worth of rhetoric, nor does it elevate those who attain high favor without it, for the intrinsic worth of a thing is different from the casual material benefit that it brings in its train.

Once I met a linguist in the company of some friends, (MS 11) and when the book **Al-Masālik Wal-Mamālik** (Roads and Kingdoms) was mentioned, the linguist deprecated the merits of the book to such an extent that he almost considered it outside the pale of the sciences. He based his argument on the utilitarian aspect, stressing the point that a knowledge of the distances between kingdoms is worthless. No wonder I was surprised! For our ambitions and objectives are different. It is said that such a difference should not lead to a quarrel, but confining one's quarrel to certain people is preferable to a quarrel with people in general.

There is no difference between him and those of our contemporaries who prefer the Persian language to Arabic, for he would say: «What is the use of the short vowel signs for the nominative and accusative cases, and the use of all the strange rules for grammatical structure? I do not need Arabic at all». This discourse of the Persian is true relative to himself only, but not absolutely.

How can I not be surprised! He (the linguist) is expected to rehearse the words of God, «Say: travel through the earth and see what was the end of those who rejected truth» (Qur'ān, Sura 6:11). And His words, «Do they not travel

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through the earth and see what was the end of those before them?» (Sura 35:44). And His words: «March forth with my servants by night, for ye are sure to be pursued» (Sura 44:23). And His words: «Now travel with thy family while yet a part of the night remains» (Sura 11:81). And all His orders for travel by day and by night, (MS 12) for the purposes of experience,

conquest, pilgrimage, and emigration. Also, the disposal of one's wealth and wordly possessions requires hard travels. Further, Gracious God relates, with His blessings, the travels of His apostles and prophets: the journey of the Two-Horned One (Alexander the Great), who came to the rising of the sun, and to its setting; the journey of Moses, God bless him, to the juncture of the two seas; the night journey of the Prophet, God bless him, from the Sacred Mosque to the Farthest Mosque, and his **Hijra** from Mecca to Medina, and his travels during his battles, and his reproaches to those who were inactive and lagged behind.

Did they travel haphazardly, and did they get used to taking poison by experience? Did they not follow specific directions on beaten roads? Did they not reckon the distances between their travel posts and those between sources of water? Have they not striven on the footpaths of guides who were able, by the grace of God, to determine, by means of the stars, the proper directions over

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the vast darknesses of land and sea? Have those travellers been other than learners receiving instructions from a man of learning, or the seekers of advice from an adviser?

It is instructive for a person who, for one reason or another, has been unable to travel, to compare between a stranger who has just arrived at a town whose lanes, markets, and streets are unknown to him, and a native of that town to whom all its features are familiar. Is there not a striking difference between (MS 13) the two men regarding their feelings of security, worry, perplexity, and confidence in movement to a desired destination? This case is analogous to that of a seasoned traveller who has full knowledge of the roads, and a stray traveller who does not know them.

If the preceding analogy does not convince one, the valuable guidance of the carrier pigeons will make one realize the value of the knowledge of directions. The magnitude of a benefit is determined by the extent of guidance and knowledge which it propagates, and the value of a person, a pigeon, or an animal, is determined by the excellence of what he does. Sometimes

straying companies of caravans resort to an exprienced donkey, among the camels, to guide them to the proper route. Thus knowledge of directions must be honored, because it elevates a donkey so that a living and speaking person calls on it for help.

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If he had heard the story about the journey of Khālid bin al-Walīd, when he crossed the desert between Irāq and Syria, and faced the danger of perishing in it, and how Khālid and his army were saved by the signs and gestures of a partially blind guide with sore eyes, he would have known that the guide had rescued groups of people who were on the verge of despair.

At a time not far distant from ours, there was, among the pilots of Sīrāf, a guide named Māfannā who was an expert in the routes across the sea. He was hired for a lot of money by a ship-owner to captain his ship to China. As he approached the gates of China, which are rivers that flow down into the sea between high mountains, the wind prevented him from entering the gate that leads to his destination, the port of Khānfū. (MS 14) So he changed his course and sailed for another gate that leads to a port other than Khānfū, but the owner of the ship demanded that Māfannā should go back to the high sea and sail for Khānfū. Māfannā warned him of the dangers of the gate which he had

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avoided, but the ship's owner insisted, and the ship was returned to the sea, where it was wrecked by a violent storm. Māfannā, however, got onto a board which made him float, but he stayed in the sea for three full days until a stray skiff, which was coming from Zābij (Java?) to China passed by him. He waved his hands to the sailors who recognized him for his fame and took him on board. They were glad he was with them, and asked him to direct their course, but he asked for wages for his services, and this demand infuriated the master of the skiff, who said, «Are you not satisfied that we have saved your life and that you share our security?» Māfannā retorted, «I would not guide you unless you pay me, for entering China in this deplorable condition is

like death to me». Then the master of the skiff said: «If you do not guide us I will return you to your former condition».Māfannā replied: «Have it your own way». So they cast him out onto his board and sailed on, but they drifted continuously in utter confusion until they perished. Māfannā remained in the sea for two more days until another stray skiff (MS 15) passed by him, and was picked up by the sailors who enquired about his adventure. After they had listened to his story, they asked him whether he was willing to pilot them. He said that he would help, provided they paid him for his job; otherwise, he would rather return to his board. So they paid him two hundred **mithqāls** of gold, and he assumed control at the helm. Then he dropped the **buld,**

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a heavy piece of lead for sounding the depth of the sea, and the protrusion of mountains rising from its bottom. He hauled a piece of clay from the sea bed, and on smelling it he was able to identify his position, and then sailed safely on the proper course for his destination.

Suppose a person can do without this kind of knowledge, because he is a laggard and a man of inaction. Are not people instinctively anxious to know what is hidden from them, and to solve their mysteries? You even find mischievous boys and hoodlums greatly interested in stories. Also, affluent people find relaxation and comfort on listening to news and gossip told by night, after they have became bored by their amusements. It is for this reason that histories were written, and stories of ancient peoples and lands were narrated. The latter, however, exist in the present, and the former is not in it; therefore the latter is of superior interest. Had it not been for the hardships of travel, and people's insuperable hindrances, most people would have liked to travel extensively and witness (MS 16) the kingdoms in different parts of the world. There is hardly a person who would not impatiently desire to witness events, unless he suffers from a mental or physical handicap which compels him to be patient and to suppress his desires.

Let us put aside all that, and ignore the recalcitrant, and let

us point out the great need for ascertaining the direction of the
qibla (the direction of Mecca) in order to hold the prayer which
is the pillar of Islam and also its pole. God, be He exalted, says:
«So from whencesoever thou startest forth, turn thy face in the
18 direction of the sacred Mosque, and wheresoever ye are, turn
your face thither» (Qur'ān, Sura 2:150). It is also intuitively
known

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that this direction varies with the place at which the direction
of the Kaᶜba is to be determined. This is witnessed in the Sacred
Mosque itself, and should be more evident, when considered
from other places. If the distance from the Kaᶜba is small, its
3 direction may be determined by a diligent seeker, but when the
distance is great, only the astronomers can determine that
direction.

Every challenge calls for the right men. They (some re-
searchers) have determined the longitudes of cities, which are
the measures of their eastern or western displacements, and their
latitudes, which are the measures of their northern or southern
6 displacements. They have done this in accordance with the
fundamental propositions of astronomy, based on the motion
of the plumb line towards the center. These people were highly
satisfied with themselves, because they were able to delve into
such minute details of a science, and thought that they were
masters (MS 17) of the whole science, and not only of its sources
9 and principles. However, when they were asked to determine
the direction of the **qibla** they were perplexed, because the
solution of this problem was beyond their scientific powers. You
see that they have been discussing completely irrelevant phe-
nomena: like the directions in which the winds blow, and the
ascensions of the lunar stations.

Even the professional astronomers find it a difficult problem
12 to solve, and so you can well imagine how difficult it is for the
non-astronomer! Most surprising of all these people is the man
working on the determination of the local meridian, who thinks
that it is the same plane all over the earth, and adds to this
another proposition, that the sun culminates at the zenith of

15 Mecca. He even composes a syllogism for it, and says: «The
time for meridian crossing is the same all over the earth, and
the sun culminates at the zenith of Mecca during its meridian
passage. Hence a man facing the sun at meridian passage is
turning his face towards Mecca.»

18 I have no patience with this logician, because he built up
his syllogism on two premises: the first of which is false, and the
second is a particular premise, but he has considered it universal.
It is useless to argue with such a man,

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because he is completely ignorant of the science of astronomy,
but we shall follow his chain of reasoning and ask him about the
application of his syllogism at Mecca itself. Why is it that the
qibla at Mecca is not on the local meridian? Why is it that at
3 places which (MS 18) are distant one mile east, or one mile west
of Mecca, people do not pray by turning their faces to the
meridian which is the same meridian for such places? To him
it is the one meridian in reality, but to the astronomers it is the
one meridian empirically.

6 Among those working on this problem, he who solved it by
observation of the pole, known as Capricorn (?), is nearest to the
truth, because its constant position gives the direction of the
course. If a man goes backwards on this course, for distances not
9 too long, his face will be turning towards the Kaᶜba, or nearly in
that direction. God says: «Turn your faces in that direction»
(Qur'ān, Sura 2:144). For an accurate determination exists in the
mind only, and so the door is left open for an actual determin-
ation by a diligent and expert researcher.

12 If the investigation of distances between towns, and the
mapping of the habitable world, so that the relative positions of
towns become known, serve none of our needs except the need
for correcting the direction of the **qibla,** we should find it our
duty to pay all our attention and energy for that investigation.
15 The faith of Islam has spread over most parts of the earth, and
its kingdom has extended to the farthest west; and every Muslim
has to perform his prayers and to propagate the call of Islam for
prayer in the direction of the **qibla.**

I do not think that my work on a correct determination, or my exposition of the methods for a correct determination, will not be rewarded (MS 19) in this world or in the hereafter. Once, I had the intention

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to glean the information provided by the method of Ptolemy, in his book, the Geography, and by the method of al-Jaihānī and others, in their books on **al-Masālik** (i.e. roads), for the following purposes: the collection of data, the clarification of obscurities, and the perfection of the art. Therefore I presented the corrected distances, the names of places and towns as reported to me orally by those who had covered those distances, and had seen those places, after carefully comparing and sifting the evidence by reporters and witnesses. To serve this purpose, I have not withheld any requested money, or the exercise of influence, and I constructed a hemisphere of diameter ten cubits to enable me to derive the longitudes and latitudes from the distances, because the calculation of these coordinates would have been too long with the limited time I had at my disposal. I kept a record of my derivations, and did not rely on committing them to memory, because I was sure at that time of my safety and security. However, when my calmity befell me, it destroyed what I have mentioned, together with my other researches, and all have vanished, «As if it had not flourished the day before» (Qur'ān, Sura 10:24). If God be pleased to decree my return, and He is Omnipotent, I shall not hesitate to complete that work.

Now I say: If by adducing rational proofs (MS 20) and true

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logical syllogisms, we can conclude that the world was created, and that the parts of its finite period, since its creation and existence, have had a beginning, we can not by such proofs deduce the magnitudes of those parts, which will enable us to determine the date of the creation of the world.

Let us consider the syllogism with the following composition: the body can not be divested of temporal accidents, and

all that can not be divested of accidents is temporal; therefore the body must be temporal and not pre-eternal (?), and an accident to a body has been proved by the first figure (proposition?). But a succession of accidents cannot be infinite, for this necessitates a pre-eternity of time, and that is impossible. For if we say that the past parts of time, i.e. periods, exist and are denumerable and cumulative, and all that exists is denumerable, beginning with the number one and ending with a finite number, then time

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must have a beginning at an assigned moment and must be finite, and it has been shown by the first figure that time has a beginning and is finite.

However, a knowledge of the parts that constitute the whole, I mean: years, months, and days, and their magnitudes, cannot in any way be realized by minds reasoning by (MS 21) syllogisms. It is possible for the beginning of time and the creation of the world to precede any assigned moment that we may fix, by an instant, or by a thousand thousands of years which are denumerable and finite. It all depends on how much truth there is in what one hears about this matter, for the Book of God and the true monuments have said nothing about it at all.

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Again, those with a book of divine revelation, like the Jews, the Christians, and others like the Sabians and the Magians, have all agreed about dating events by the Era of the Creation of Mankind, but they differ greatly in their estimation of the duration of that era. They have not referred to the Era of the Creation of the World, except in the opening two verses of the Torah, which have the following content but not the exact wording: «In the beginning God created the heaven and the earth. And the earth was without form and void, and the spirit of God was moving upon the surface of the waters» (Genesis I: 1,2). They considered that to be the first day of the week in which the world was created, but that was a period of time which cannot be measured by a day and a night, for the cause of these

periods is the sun with its rising and setting, and both the sun and the moon were created on the fourth day of that week. How is it possible to imagine that these days are like the days of our reckoning! (MS 22) The Qur'ān says: «A day in the sight of thy Lord is like a thousand years of your reckoning» (Sura 22:47). In another verse, God says: «In a day the measure whereof is as fifty thousand years» (Sura 70:4). Thus it is obvious that we cannot estimate that period with our method of reckoning, and that it is unverifiable since the beginning of creation.

Though the Torah related that the first man was created on the Friday of the week of creation, God Almighty has related that his angels had said: «Wilt Thou place therein one who will make mischief therein and shed blood? Whilst we do celebrate Thy praises and glorify Thy holy name» (Qur'ān, Sura 2:30).

We do not know of the conditions of creation, except what is observed in its colossal and minute monuments which were formed over long periods of time, for example, the high mountains which are composed of

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soft fragments of rocks, of different colors, combined with clay and sand which have solidified over their surfaces. A thoughtful study of this matter will reveal that the fragments and pebbles are stones which were torn from the mountains by internal splitting and by external collision. The stones then wear off by the continuous friction of enormous quantities of water that run over them, and by the wind that blows over them. This wearing off takes place, first, at the corners and edges, until they are rubbed off and the stones finally take an approximate spherical shape. (MS 23) As a contradistinction to the mountains, we have the minute particles of sand and earth.

When soft fragments and pebbles accumulated in the beds of valleys, they became a compressed mass; then sand and earth mixed thoroughly with it and formed a combined mass, and when torrents of water flowed over it, that mass became embedded in the deep bottom after it had been on the surface of the earth. (The mass) was petrified by the cold, because the petrification of the interior of most mountains is caused by low

temperature. This is why stones melt under the influence of heat, because what is formed by low temperature dissolves by heat, and what is done by heat is undone by low temperature. Whenever we find a mountain formed from such softs stones, and there are many such mountains, we know that it has been formed as we have described, and that it had sunk once and has risen once more.

All those changes are necessarily of long duration, and their causes are of unknown nature. They have influenced man's habitation and social development over different parts of the earth; for when big masses of the earth move from one side to another, their weights move with them, and the earth can not keep its stability, unless its center of gravity remains at the center of the universe. But its center of gravity varies in position with the variation of the distribution of mass over its surface, and so the earth must adjust the distribution to keep its (MS 24) stability. Now,

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the distances of different regions from its center of gravity are not invariable over long periods of time. If the land adjoining a district had risen up, or had been sunk, then the waters in that district would have been diminished, the sources would have been sunk, the valleys would have been made deeper, and the district would have been rendered uninhabitable. Its inhabitants would have moved to another district; and people would have ascribed that destruction to old age in the former district, and the building up of the desolate land in the latter would have been ascribed to its youth.

Abū al-ʿAbbās al-Īrānshahrī related that he had seen in a castle, known as the 'White', at a distance of one farsakh from Shīrjān, a town in the district of Kirmān, the stems of date-palms which used to grow there, but the climate of the locality grew colder and the palms died and withered, and that at his time there were no palms within a radius of twenty farsakhs from the castle. He also added explicitly that, when the level of that locality rose, many brooks and rivers, that had been flowing in the adjoining land, were sunk.

Similarly, a sea is changed to dry land, and dry land to sea over long periods of time. If these periods have passed before the creation of mankind, then they are unknown, and if after that epoch, there are no records of them. For reports are usually discontinued after a long period of time, and those about very slow events, in particular, are remembered by educated people only.

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This steppe of Arabia was at one time sea which was later upturned, (MS 25) and the traces of that sea become evident on digging for wells and springs, because the desert then unfolds various strata of earth, sand, and soft pebbles, intermingled with pieces of pottery, glass, and bones, which could not have been buried there intentionally. Again, a variety of stones is excavated which reveals, on breaking up, definite sea products: shells, cowrie shells, and what are called «fish ears». These products will be found, either fully preserved, or in a state of complete decay, and in the latter case they will have left their figures completely imprinted in cavities in the stones. Such remains can also be found in (the city of) Bāb al-Abwāb on the coast of the Caspian Sea. The durations and dates of such transformations are completely unknown.

The Arabs have lived there since the days of their forefather Yuqṭān, but it is possible that they lived in the mountains of the Yaman, when the Arabian desert was a sea. Those were the ancient pure Arabs who dwelt in the Yaman, behind a dam between two mountains, where the water rose to the top of the mountains, and enabled them to cultivate two rows of gardens: one on the right, and one on the left of their dwelling places. But a torrential flood destroyed the dam; their habitation became desolate, and their two rows of gardens were transformed to two other gardens, «Producing bitter fruit and tamarisks, and some few Lote trees» (Qur'ān, Sura 34:16).

We find the like of these stones, with «fish ears» in their middle, in

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the sandy desert between Jurjān (MS 26) and Khwārizm. This

desert was like a lake in the past, because the course of the Jayḥūn (the Oxus), I mean the river of Balkh, ran through that desert to a town called Balkhān, on the Caspian Sea. Ptolemy relates in his book, the Geography, that the mouth of the Oxus is at the sea of Hyrcania, i.e. Jurjān. The time that has elapsed since the days of Ptolemy is about eight hundred years, and in those days the Oxus ran through the region, which is now a complete desert, between the towns of Zamm and Āmūya up to Balkhān, giving rise to prosperous towns and villages, and finally emptying its water into the sea, between Jurjān and the land of the Khazars.

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It happened that an obstruction blocked its course, and diverted its water towards the land of the Ghuzz; then its course was obstructed by a mountain, now known as the Lion's Mouth, but the Khwārizmians call it the Devil's Barrier. Its water gathered before the barrier, and traces of the waves that lashed its summit are still visible. But when the pressure of the water mounted, and overcame the resistance of the crumbly rocks, the water rushed through the barrier for a distance of a day's journey; then it swerved to the right towards Fārāb, following a course which is known at present by the name of Fuḥma, and people built up on its banks over three hundred towns and villages whose remains are extant.

Later, this second course was also obstructed like the first, and its water was diverted to the left, to the land of (MS 27) the Bujnākians, to a course known as the valley of Mazdubast, in the desert between Khwārizm and Jurjān. It gave prosperity to many regions for a long time,

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but also ruined other regions whose inhabitants emigrated to the shores of the Caspian Sea. These emigrants are of the race of Allān, and that of Al'ās, and their language is a mixture of the languages of the Khwārizmians and the Bujnākians.

Then all the water flowed towards Khwārizm, where only minor streams had flowed previously, after trickling through a barrier of rocks which stands now at the start of the Khwārizmian

plain and, on dislodging the rocks, the water poured through them, flooded the region and turned it into a lake, with the barrier as its initial boundary. A considerable amount of clay was carried with the water, because of its enormous quantities and its very rapid flow, and hence a mass of earth was deposited at the start of the lake. As the accumulated deposits gradually hardened, the lake receded from its initial boundary until the whole of Khwārizm emerged. The lake continued to recede until it reached a mountain which it could not overcome, so it swerved to the left, to the land inhabited at present by the Turkomāns. This lake is not too far from that which had existed in the valley of Mazdubast, and the latter has been converted into an intractable swamp which is known by the Turkish name Bakhīz Tanqazī, i.e. the Sea of the Virgin. (MS 28)

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Ibn al-ᶜAmīd related in his book, **Fī Binā' al-Mudun,** (on the Construction of Cities), that an earthquake took place in Rūyān, not long ago, which made two mountains collide and tumble down, and that the debris of the collision blocked the courses of the rivers which ran between them, and that the waters of the rivers receded and formed a lake. This is what usually happens when the water has no outlet, like the Dead Sea which is formed by the water of the Jordan River.

He also quoted from the histories of the Syrians that in the year three hundred and thirty-eight of Alexander, the second year of the Caesar Justinian, an earthquake took place in Antioch, and was accompanied by a fault in the earth crust, and that a mountain above Qalūdhia (Clodia) had crumbled, and its debris fell into the Euphrates. The river was blocked; its water rose and flooded the area, causing a great deal of destruction; then the water swerved round its barrier, found an outlet for itself, and returned to its normal flow.

Aristotle related in his book, al-Āthār al-ᶜUlwiya, the Meteorologica, that the land of Egypt, in ancient times, was completely covered by the Nile which made it look like a sea; then the Nile water receded; the emerging land hardened gradually and was inhabited, and many towns sprang up which

were full of people. But the Egyptians of today

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are ignorant of the art of construction. In ancient times Egypt was called Thiba (Thebes) after the name of one of its upper cities which was early constructed and inhabited; it is not (MS 29) the great city of Minf which is now called Mimfiyas (Memphis). The poet Homer, who is comparatively more recent than the early Egyptians, also calls it Thebes.

When the land of Egypt was a sea, the kings of Persia, during their partial domination over Egypt, wisely decided to excavate between it and the Red Sea, and to construct a canal which would connect the two seas; so that a ship could be navigated from the sea in the west to that in the east, and all this was done in good faith, to promote the country's welfare. The first to render this service was the king Sāsṭrāṭis, (Sesostris) and the second was Darius. They both ordered the excavation of a long canal which still remains, and the water of the Red Sea enters it with the tide and exits with its ebb. However, when they measured the depth of the water in the canal, they decided to drop the project because the level of the Red Sea was found to be higher than that of the river of Egypt, and the risk that the sea water might spoil that of the river. The canal was completed by Ptolemy the Third, and the risk mentioned was eliminated by the expert advice of Archimedes. But the canal was later filled up with earth by order of one of the Roman kings, to prevent the Persians from coming to Egypt through it.

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Again, the desert known by the name of Karkas Kūh, between the districts of (MS 30) Fārs, Sijistān (Seistan), and Khurāsān, is full of the ruins of decayed settlements which Ptolemy calls Qarmania the Deserted, i.e. Kirmān the Deserted. The Persians relate that it was the most prosperous country, for it had a group of over a thousand powerful water fountains whose sources were in the neighborhood of Sijistān, and that the Turk Afrasiāb destroyed them. Thus water was denied to that area, and consequently it fell into ruins, but what remained of that

6 water had flowed and formed Lake Zarah which did not exist
before.

In Syrian regions and other waterless deserts, where no
plants and animals can exist, you may see ancient remains which
prove conclusively that those lands were habitable, and that this
would have been

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impossible, if those regions had no water, which later must have
ceased to flow. You would also see traces of civilization in the
swamps of Basra which were not formerly on the course of the
3 Tigris, but were later flooded by it.

Abū al-ᶜAbbās al-Īrānshahrī related that a canal was dug
out in the district of Busht, from the borders of Nīshapūr, and
6 that the trunks of three willow trees, which had been sawed with
a saw, were found in the canal at a depth of over fifty cubits. It is
obvious that, since the trees were cut on the surface of the earth,
the time during which that depth of earth had been compressed
is too long to be assessed accurately. Also, one should not be
surprised that the wood has been preserved for so long, because
9 when it is kept away from a place which is exposed to extremes of
heat and cold that alternate through the year, it is more likely
to be preserved (MS 31) longer.

Also, that log in Jurjān, which emerges each year from a
source of water with its roots sticking in it and its top revolving
at the edge of the fountain. The people of Jurjān have a great
deal of superstition about it, and they think it is something great.
12 It is really nothing more than a willow tree whose ground was
split by an earthquake. The tree fell into the fault, and then earth
was heaped up over it, but the source of water in the fault did
not give the tree sufficient buoyancy to lift it up, and so its
branches rotted and fell down. In the Spring, however, the water
in the source increases,

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and the tree emerges because its buoyancy has increased, but it
does not completely emerge from the water, because some of its
3 roots still cling to the bottom of the source. People who have

dived into it, and touched its walls, have reported that it re-
sembles a **tannūr** (an oven like a truncated paraboloid of revolu-
tion) which is filled during the days of flood, and that the log
sinks to the bottom of the **tannūr** when the water goes back to
its normal level. None of the people in that region can tell you
the date of its story.

6 It is well known that civilization demands water, and that
it shifts in pursuit of it, because the former is dependent on the
latter. Aristotle related in his book, the Meteorologica, that some
ancient people held the view that the earth was moist, and that
the sun and the moon evaporated the moisture from some regions
9 until they dried up, and the vapor produced winds which cir-
culated in the air, and that the moisture that remained has formed
the sea which will diminish, steadily (MS 32) decrease, and
finally become dry land. This view, though systematic, apparent-
12 ly contradicts observed natural phenomena, but if some inter-
pretation is provided, it can explain the realities of Nature. For
it has been established in fundamental astronomy that the earth
is spherical, amid a spherical universe, and that the plumb line
naturally moves towards the center from any initial position or
15 direction. This makes it clear that the surface of the water must
be spherical, except for irregularities of surface produced by the
waves, because of lack of cohesion between the particles of water.

It is also known from observation that the natural position
of dry land is below that of

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the water, for soil precipitates in water, and that the penetration
of water, falling vertically into soil or earth, is due to the rarefac-
3 tion of the air, and the natural tendency of the water to settle
below the air which permeates the space between the agglomerat-
ed particles of the soil.

It is also known that if parts of the earth are disrupted by
violent force, then they move about the center and that if a
disruption takes place, then water surrounds a disrupted part
equally on all sides.

6 What follows is the state of things which was prevailing
at the beginning of creation, as related in the Torah; I mean the

movement of the breath of God on the surface of the water, when the earth was void and desolate. The same state was reported in the Revelation, where God Almighty says: «And His Throne was over the waters» (Qur'ān, Sura 11:7). When God Almighty intended the creation of mankind, He purposely designed the creation of the earth at first, and gave it the consolidating force to evolve (MS 33) its natural shape, I mean that which is truly spherical. He also elevated parts of the earth above the water, and this made the water run down into parts of the earth which were sunk, because of the elevation of others. He called the water that gathered in a depression a sea, and gave it the taste of salinity. This salinity, according to Thābit bin Qurra, prevents the water from getting foul, and eliminates putrefaction which would be disastrous to His intended creatures. The sea was also intended to be a reservoir of water for man's special needs, and as the lives of both man and animal, which is put in man's service, are dependent on fresh water, and as his habitation is far away from reservoirs, so God Almighty has designed the continuous motion of the sun and the moon, and commanded them both to produce motion in the water, to evaporate it, and to lift its vapor upwards. Further, the elevation of parts of the earth above the water was designed as a connecting link between

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the earth and the air, and water was made to possess the properties of permeation and diffusion, but all that is hardly possible without the influence of heat.

At the time of creation, when He put the celestial orbs in motion, the air adjacent to them was set on fire; so He set the stars in motion to conduct the heat to the center, and He varied their velocities with the variations of their declinations and their distances from the earth. Thus the motions are not of uniform and constant speeds, but are of different periods and cycles, because Nature becomes weary with exertion, and all that labors under strain requires rest.

Then He commanded the winds (MS 34) to drive water vapor, in the form of clouds, to desolate and waterless lands; so that its rain in those lands will refresh and sustain the lives of

animals and plants over there, and its rain on the mountains will penetrate and accumulate deep inside them, or will remain on their tops in the form of snow. Further, the accumulated water will form rivers which will carry it back to the seas, but their courses will run by the dwelling places of peoples and animals, who will use the water for drinking and other utilities. These benefits could not have been possible, if the solute in sea water were other than salt; because vapors of solvents, except the vapor of a saline solvent, carry the tastes of solutes dissolved in them. For example, bitter water is injurious to animals; sweet water is more readily fouled than fresh water; acid water is repugnant, makes surfaces hard and rough, and reacts so vigorously that it changes whatever comes into contact with it, and it is sufficient to mention its action on iron and similar metals. Glory be to God, Most Omnipotent, Most Wise!

Hence the statement (related by Aristotle) can be explained as follows: The sea always evaporates, but its location may become dry only if its water shifts to another location, and the idea of its total disappearance is so impossible that it should be completely disregarded, since it would lead to the destruction and extinction of all kinds of animals, and the negation of perfect

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design, in addition to the abrogation of one of the basic four elements, which is water.

Some people thought that there is dry land in the south similar to the north, and that there are also people and animals. (MS 35) Aristotle, however, did not assert this; he merely considered it a possibility. He said: «If a place like that exists, and its position relative to the South Pole is analogous to the position of this place relative to the North Pole, then the winds and all other vestiges there must certainly be similar to what we have». What a fine thing he (Aristotle) has said! The proximity to the pole implies distance from the equator, and this proximity, or that distance, is the main influence on the climatic conditions of cities. It is like the influence of the sun during its diurnal rotation; the influence of its zenith distance at a place is analogous to the influence of distance from the equator at that place.

12 He also stated conditionally: «If a place there (in the south) exists, analogous to this (place in the north), i.e. of the same nature of terrain, the same altitude above sea level, and at an equal distance from the equator, which determines climatic
15 conditions, then the influence of heat and cold at that place must be identical with their influences at this place, and all other meteorological phenomena which are influenced by heat and cold should be identical.

Further, he (Aristotle) made no mention of people or ani-
18 mals, because this depends on observation or on a truthful report. If we notice the distribution of people and civilization where we live, in the inhabited parts of the world which are on (MS 36) the same circle of latitude, where

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climatic conditions are identical and means of civilization are available, where causes of diseases are eliminated and atmospheric conditions are the same, then we can not attribute the absence of people in some places and their concentration in
3 others to anything other than freedom of choice, or to agreement with others, or to the possibility that no human beings ever set foot on those uninhabited places. But the elevation of land above the water in the southern quarter, diametrically opposite the northern quarter, is possible if the spherical shape of the earth has
6 been distorted to a cylindrical shape; so that the empirical sphere shall envelop both the spheres of earth and of water, and the center of gravity of the Whole shall be at the middle of the axis of the cylinder, in order to preserve the equilibrium of the weight. However, a portion of the spherical surface can be removed,
9 and cavities then produced will be partly filled with water from the surrounding sea, while the tops of those cavities will remain above the water, and so the water will be all over the land, except that portion from which the mountains are made.

Some people have argued as follows: As the sun attracts
12 moistures and evaporates them to dryness, and as it lifts up from the seas the lightest and the freshest, then what remains of sea water must be dense salt water which is attracted by the sun, but is inseparable from the water. We can notice the difference

between light moisture and dense moisture if we put a drop of
15 each liquid on a surface that is being heated by the sun. It will evaporate (MS 37) the light drop to dryness, and nothing will remain in its place, except its color if it has one, but it will gather the dense drop at its center, and evaporate to dryness its light moisture, and the periphery of the dense drop will resemble what
18 is left of the light one, but on complete dryness the center of the dense drop will protrude and point out in the direction of the sun, being attracted by it. If one likes, one can examine this on a sheet of paper with two kinds of ink: a light one, and a dense one.

They said: «The astronomers have told us that when the
21 sun, moving southwards, is at its farthest distance from our zenith, it will then be at its nearest distance to

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the earth. But we know that when it comes nearer to the earth its influence will be stronger, and hence the evaporation of fresh and light water will considerably increase, and the attraction of dense
3 and salt water to the south will also increase. Therefore most of the salt water was attracted to the south during the sun's culmination at the zenith, and so the south has become a sea, and the north a dry land.

They said: «They (the astronomers) have also told us of the motion of the farthest point in the sun's orbit, which is
6 called its apogee, along the succession of the (zodiacal) signs, and this leads us to think that when the sun's perigee is in the zenith of the northern quarter, then the sea should shift to it and dry land should appear in the south.

In what they have said, there are several aspects to be
9 considered. First, the perigee of the eccentric orbit, or the epicycle, will be at the zenith of a (terrestrial) parallel of latitude completely encircled by the rotation of the whole, and not in the zenith of (MS 38) a single region, as they claimed, and similarly for the apogee in the north. So if the theory which they have
12 given is accurate, then the whole parallel of latitude and its vicinity should be a sea whose maximum rise should revolve with the sun, as the maximum tide of the seas revolves with the

moon. But if it is said that this is actually what takes place, and that there is no dry land in the south opposite the inhabited northern quarter, then (according to their theory), the northern parallel of latitude, and its vicinity, which is under zenith transit of the sun in apogee, should be completely dry land, whether inhabited or uninhabited. Reality, however, contradicts this deduction.

Secondly, the astronomers have not reported about the sun's eccentric orbit, or its epicycle, the way they have reported about its spherical shape and magnitude, i.e. by way of observation. They found it necessary to impose such an orbit in order to explain the sun's observed variable speed, though it is impossible to prove that this variability is an intrinsic property of the sun, and if there had been no observed variable speed no variable displacement would have been imposed on the sun itself. Abū Jaʿfar al-Khāzin stated, in one of his treatises, that it is possible to imagine

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a variable speed for the sun about the center of the universe, and a uniform speed about a different center; as it was possible to imagine a variable speed for the center of the moon's epicycle along the circumference of the deferent, and a uniform speed about the center of the Whole, and as it was possible also, in the case of the planets, to imagine variable speeds for the centers of their epicycles, on the circumferences of their (MS 39) eccentric deferents, and uniform speeds about the centers of their equants. If all that is possible, then a very serious criticism can be levelled at the moral integrity of those people, until they can account for the apogee and the perigee on grounds other than the sun's variable speed.

Thirdly, if the sun's proximity in the south and its zenith culmination there are the causes for the attraction of the water to the south, then the transfer of water should increase the weight of the southern hemisphere and displace the center of gravity of the earth southwards, and thus the land in the north would be at a greater distance from the center of gravity and the land should rise. This was pointed out by Ibn al-ʿAmīd. Also, when the sun moves north and the south gets cooler, the earth should go back, wholly, or partly, to its former state, and thus the earth should have been in continuous oscillation, with the water rising at one time and falling at another.

Fourthly, the motion of the apogee has been confirmed by some observers and rejected by others. I am not denying the existence of this motion, but simply reporting the opinions of different observers about it. We related previously the story of the creation of the world, and the various periods of its date. It is quite possible that the time since creation has not been sufficient to complete a cycle of the apogee, or a part of it, and it is also possible that several cycles have already been performed, and that more will be performed in the future. I have gone into some details to inform the reader about this subject, but I have no proofs to offer.

Aristotle's disregard (MS 40) of secondary influences on a place is very fine, indeed! He rejected such influences as in his conditional statement:

> If we desire a natural approach to probe this subject, we should imagine the earth to be dispossessed of mountains and seas, so that the different regions shall be influenced naturally and regularly by the sun's zenith distance at culmination.

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We also postulate that lack of civilization in the South is due to the sun's culmination there when it is in perigee, and that its excessive influence is due to proximities of two kinds: I mean proximity to the zenith and proximity to the center of the earth. As the years roll on, the sun would be in perigee near the winter solstice, and the southern regions, most influenced by its heat, are those which fall under zenith culmination, when the sun is in the parallel of this solstice. We know that its zenith distance at transit, in the middle of the first climate, would then be forty degrees, and that people living there do not suffer from its heat. Therefore the southern region at a distance of forty degrees south of the parallel of the winter solstice, i.e. the region of latitude sixty-four degrees south, will have an atmospheric condition, during that time, which is comparable to that of the

middle of the first climate, and so it is possible for animals to
exist in that region.

Now, let us consider the case of that region when the sun
is in apogee, (MS 41) near the parallel of the summer solstice.
When the sun revolves in this parallel, its zenith distance during
12 transit in that region which we have defined in the south would
be eighty-four degrees. There is no inhabited region in the north,
displaced by so many degrees from the parallel of the summer
solstice, with which we can compare the atmospheric conditions
15 of the region, because the displacement of the north pole from
that parallel is only sixty-six degrees, one quarter, and one sixth.
Therefore we shall have another consideration. The region,
where the sun's farthest zenith distance at transit is eighty-four
degrees, has a latitude of sixty degrees, and regions of such a
18 latitude, or much less, are not habitable, because of the extreme
cold which is caused by increase in the sun's zenith distance,
even when it is at its nearest distance to the earth. Surely the
cold will increase when an increase in the earth's distance is
added to an increase in the zenith distance.

21 So, by analogy, there must be an alternation at a locality
of latitude sixty-four degrees south; it will have the heat of the
middle of the first climate if the sun is

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in perigee, and the cold of a locality of latitude sixty to the north
(MS 42) if the sun is in apogee. The amount of that heat can
be tolerated by the animal genus, but the amount of cold would
3 be destructive. Further, in a place of lower latitude, towards
the south pole, the cold increases, but the heat tends to be
medium, and in a place nearer to the solstitial parallel the heat
increases, but the cold diminishes, and so animals are more
6 likely to exist there, because if the equatorial region is habitable,
the place of latitude forty-eight degrees south has the same
amount of heat as that region, and the same cold as a region
of latitude forty-eight degrees north. But the law of Nature
prohibits the continued existence of an animal in this region,
9 because it can not resist the alternations of excessive heat and
cold, which are comparable to change of weather from autumn

to spring. For this reason, as well as others not to be excluded,
the alternations would be sickening and annihilating.

Civilization in the north is made possible by balanced
12 climatic conditions, for zenith culmination produces an increase
in heat, and bigger distance from the earth produces a diminution
of it. Thus the effective resultant is far from extreme, and it is
either a happy medium, (MS 43) or near it.

15 In the south, however, the extreme of zenith culmination
is added to extreme proximity to earth, and no balanced effect is
reached. All this is designed by the All-Wise. It is not fortuitous
or haphazard, for He placed the water where civilization is not
possible because of unfavourable climatic conditions, and made
18 the land emerge where habitation and civilization can flourish.

Ibn al-ᶜAmīd stated that, if the south were made of land,
the winds that blow from that direction would have been very hot
and destructive, but because it was made of water, the moisture
21 has eliminated that menace. We can point out to you that the
winds which blow from waterless deserts are

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excessively hot, and that the land of Egypt becomes very hot, and
that of Shīrāz very cold, because the deserts of the Sūdān lie to
the south of the former, but the Persian Gulf lies to the south of
the latter.

3 We discussed previously the state of the earth when a
transfer of its parts takes place along its surface, and the conse-
quent transfer of the intermediary parts lying in the direction
of the drift, and that the entire earth must necessarily move in
this direction, and that the natures of regions and their atmos-
pheric conditions are subjected to change because of the changes
6 of distances of those regions from the center of the Whole. Now I
say that this movement - though it is haphazard, irregular,
sensibly small over a sensible period of time, and takes place
gradually along a diameter of the Whole - may be rotational about
the center, or a resultant of (MS 44) the two motions whose
9 direction may be one of the four cardinal directions or an inter-
mediary direction, or it may be an impulsive jerk because of a
sudden transfer of the weights from one position to another. Such

a movement would affect adversely a fundamental principle of astronomy, like the sun's declination, though its amount in the celestial sphere remains the same. Its critical test, however, would be the altitudes of the two solstices; for if that movement happens to take place between two observed solstices, it may increase or decrease the maximum declination. But the frequent and successive observations conducted so far have not detected that accidental defect. Latitudes may be changed sensibly by that movement, and even the direction of latitude may be affected; or a dangerous displacement may be produced which can cause havoc and destruction. Therefore latitudes should be continually observed and examined. Apart from that change, the movement may slightly affect the parallax.

The adverse effect of that movement on longitude is insignificant, if the movement is to the east or to the west, but if it is to the north, or to the south, its adverse effect would be considerable, because when similar arcs are exchanged, their difference becomes apparent and their difference in magnitude becomes obvious.

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Let it now be known that, in this discourse, my general purpose - though I have digressed from its track - is to (MS 45) explain, for specified places on the earth, valid methods for the correction of the following data: (a) The east, or west, longitude of a given place. (b) The north, or south, latitude of a given place. (c) The distance between two specified places. (d) The direction of one place relative to another. My particular purpose, however, is to determine these data for the city of Ghazna, the capital of the Kingdom of the East, because, as a newcomer, I would like to consider it, by human reckoning, my homeland; though all true reckoning, in reality, is made by God only. In Ghazna, as long as I am able to do so, I shall persevere in carrying on the observations and the scientific research on that which is constantly in my mind, namely, the determination of the true direction of the **qibla.** This is not a personal matter, it concerns the people of Ghazna, as well as myself, and everyone staying temporarily in the city.

I beseech God to lead to the right insight, to help to find the truth, to illumine its paths and make them easy to tread, to remove all hindrances to the fulfillment of virtuous desires, by His munificence and ample bounty. Over that which He permits, He is Omnipotent, the best Lord, the best Supporter!

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II. ON THE INDEPENDENT DETERMINATION OF THE LATITUDES OF CITIES

There are two methods: (1) By observation of the fixed stars. (2) By observation of the sun (MS 46).

The first method is divided into three classes: (a) By stars whose parallel circles are permanently above the local horizon. (b) By stars whose parallels touch the local horizon. (c) By stars whose parallels intersect the local horizon.

Further, each of these classes is divided into three groups: (i) Where the zenith is inside the parallel. (ii) Where the zenith is on the parallel. (iii) Where the zenith is outside the parallel.

From the solar method classes (a) and (b) must be excluded, because there is no civilization where the sun's parallel is included in those classes, but if required, the latitude can be equally obtained by the method of the fixed stars. Again, the solar method is applicable where the zenith is (i) inside the parallel, (ii) on the parallel, (iii) outside the parallel.

I deal first with the stars whose parallels do not intersect the local horizon. The stars in this class are called the «permanently visible», i.e. circumpolar stars. Let ABG (Fig.1) be the local meridian; BG the semi-horizon whose pole is A, the local zenith; and let point M be an intersection of the equator with the meridian, and E be the pole of the equator. So, because ME is a quadrant, and AG is (also) a quadrant, hence if we subtract EA, which is common (between them), AM will remain (and be) equal to GE. But AM is the latitude of a city where the horizon is BG, the zenith is A, and

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the altitude of the pole is EG. Hence the observed altitude of the pole E is equal to the latitude of the city. (MS 47) E is the pole of the celestial equator; it is the pole of all the parallels

because they are all parallel to the equator, and therefore it is the pole of the parallel DT. The altitude of a star revolving in this parallel varies; it increases in the east until it reaches point T. Hence TG, in both the first and the second figures, is a measure of the maximum altitude, but facing south, its measure is given by TB in the third (figure). Afterwards, as it moves westwards, its altitude decreases until it reaches point D and, facing north, its minimum altitude would be equal to DG. This altitude may be called the star's descent, and the first its ascent. It is known that ED is half the difference between the two altitudes in the first and second parts of the figure (Fig.1), and (it is equal) to half the sum of the co-altitudes which are DA and KA (read TA) in the third part. If it is added to GD, the lesser of the two altitudes, GE, the latitude of the city is obtained.

In the third part (of the figure) GD cannot be equal to BT, because a supposition of equality implies the coincidence of E with A, but T cannot fall on M, for the only parallel through M is the equator, which is a great circle that intersects the horizon, and we have supposed it to be a non-intersecting parallel.

For the computation of latitude, we observe the minimum and the maximum altitude of a circumpolar star as it crosses the local meridian, and consider the following cases: (a) If both altitudes are above the northern horizon, we find half (MS 48) the difference between the altitudes, then add this to the lesser altitude, and the sum will be the latitude of the city. (b) If the altitudes are in different

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directions, we find half the sum of the coaltitudes, then add this to the lesser altitude, and the sum will be the latitude of the city. (c) If one of the altitudes is exactly ninety degrees, we find half the co-altitude of the lesser, then add this to the lesser altitude, and the sum will be the latitude of the city; or we can add half the lesser altitude to one-eighth of a rotation, and the sum will be the latitude of the city. The proof of the last statement is as follows: The ratio of AD in the third situation to a quarter of a rotation is as the ratio of AE to one-eighth (of a

rotation), and the ratio of the difference between AD and a quarter to the difference between AE and an eighth is as the ratio of a quarter to an eighth. Therefore DG, the second difference, is equal to twice the first difference, but the second difference is the lesser altitude and the first difference is the excess of the altitude of the pole over one eighth of a rotation.

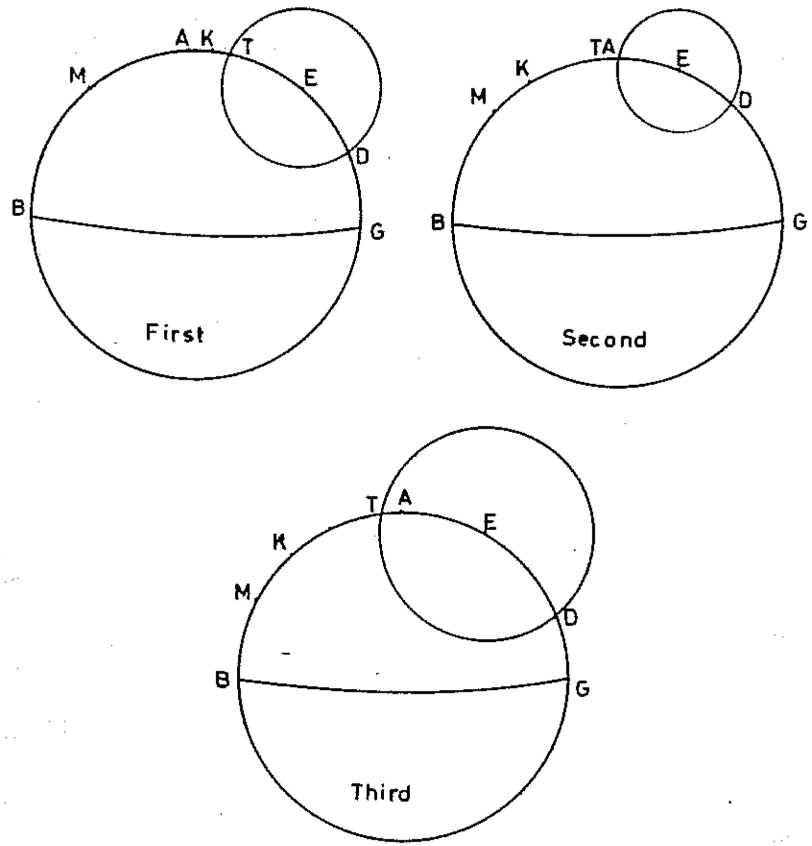

Figure 1

Further, if we add the lesser altitude to the greater, then

half this sum will be the latitude of the city. Proof: Take TK equal to GD then GDK is the sum of the two altitudes. But ET and TK are equal to ED and DG. So half of GTK, therefore, is GE, the latitude of the city. (MS 49)

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One of the observations carried out along these lines and known to me is that which was made in Baghdad by Muḥammad and Aḥmad the sons of Mūsa bin Shākir. They observed the maximum and minimum meridian altitudes of the eighteenth star in the Great Bear - it is the star at the root of the Bear's tail and next to the **Banāt** of the Great **Na^{c}sh.** They found that its maximum altitude is 60;46°, and its minimum altitude is 6;5°. If we subtract the lesser from the greater, there remains 54;41°. Now, half of what remains is 27;20,30°, and if we add this to the lesser altitude we get 33;25,30°, which is the latitude of Baghdad.

They (the sons of Mūsa) also observed at Baghdad the nineteenth star in the Great Bear - it is (MS 50) the second of the two stars, next to the **Banāt Na^{c}sh,** and on the left hind shank of the Bear. They found that its maximum altitude is 63;13°, and its minimum altitude is 3;45°. So the sum of the altitudes is 66;58°, and half this sum is 33;29°, which is the latitude of Baghdad.

They also observed at Baghdad, the twenty-sixth star in the Great Bear, at the middle of its tail - it is the middle star of the **Banāt,** and is adjacent to the star, **As-Suha.** They found that its maximum altitude is 62;3°, and its minimum altitude is 4;8° (4;48). If we add the altitudes we get 66;51°, and half this sum is 33;25,30°, which is the latitude of Baghdad.

I found in some manuscripts that the maximum altitude of this star is 62;13°.

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It follows from this that the latitude must be 33;30,30°. However, such a minute difference may be undetectable by observational instruments, and it may be a copyist's error. No dates were given to these observations in the original manuscripts, but I suppose they were made about the year two hundred forty-eight of the

Hijra, which is the year two hundred thirty-two of the Persians. God knows best.

If the star observed is one of those that rise to a maximum altitude at transit, and if it does not pass through a least altitude, as it declines westwards, but touches the horizon as it crosses the meridian, (MS 51) then half that maximum altitude is the latitude

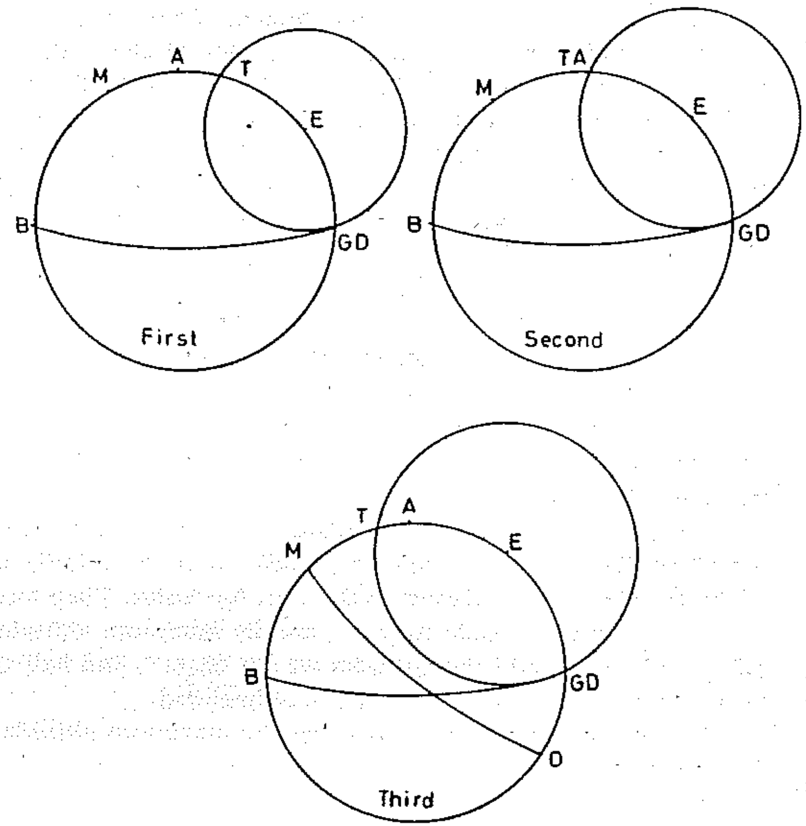

Figure 2

of the city, if the transit occurs between the zenith and the north point. This is illustrated in the first two parts of the figure (Fig.2).

But if it (transits) on the south side, as in the third part of the figure, then GE is half of GAT, and GAT is the sum of GA, a quadrant, and AT, the complement of the altitude, in order to obtain, in the third part, the celestial equator, which is MO. So OG will be the complement of the latitude of the city. But OG is equal to TM and MB is the complement of the latitude of the city. So TM and MB are equal. Hence if we take half the altitude TB, we get MB, the colatitude, and AM, the complement of the colatitude, gives us the latitude itself.

The computation is as follows: If the transit is not between the zenith and the south point, then half the altitude is equal to the latitude of the city. If on the other hand, the transit is between the zenith and the south point, and if the co-altitude is added to ninety, or the altitude is deducted from one hundred and eighty, then half the result, in either operation, will give the latitude of the city.

68 (MS 52)

If we can never find a star that is always visible, and whose parallel does not intersect the horizon, then we are on the equator. As evidence of that, when a star transits at the zenith, its rising on the right and its setting on the left are actually on the diameter.

If the star observed is one of those whose daily paths intersect the horizon, I mean that it has a rising point in the east and a setting point in the west, and since the position of the observer can be regarded as the center of the Whole, so let it be E (in Fig.3); and let BG be the meridian line, ABD the star's daily path, and AGD the line of intersection of its plane with that of the horizon. Let us prepare, of any solid material we like, three straight and equal rods: EK, EL, and EM, and let us observe the star at any three

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arbitrary times. However, the wider the intervals between them are, the more reliable will be the result of the observation. Let Z, H, and T be the star's (MS 53) positions in its daily path at the three chosen times. We collect the three heads of the rods in free pivots at E and, at each chosen time, we fix one of the rods in the

line of sight of that same star, either by placing the observer's eye straight along on the rod or, as usual, by the use of two perforated visors. If we do that, EK will be collinear with KZ, EL with LH, EM with MT, and the three rods will be generators on the surface of the cone whose vertex is the center of the Whole and whose base is on the circumference of the parallel. Because the rods are of equal length, their tips, I mean K, L, M, will be on the circumference of a circle which is parallel to the daily path ABD. We join K and L with a thin strong thread and adjust a ruler at the tip L (along LK), so that L can be moved along the ruler and it does not obstruct it (L) from hitting the plane of the horizon. We then pass the ruler to thread KM and, without sliding the thread, we move the ruler until it meets the horizon at S. Point S is in the plane of the circle (KLM) and must lie on the line of intersection of its plane with the plane of the horizon, which is therefore a line parallel to AD. Let us construct SF (in the plane of the horizon) orthogonal to EG, and drop LO perpendicular to the plane of the horizon. From O, the foot of the perpendicular, we draw OF parallel to EG, and we join LF. Angle LFO is (MS 54) a measure of the colatitude of the city, because LF is in the plane of the circle and is parallel to the line joining G to the midpoint of the arc AD, and the triangle FLO is similar to the triangle formed by joining G to the extremities of the perpendicular dropped from (B) the middle of the parallel upon the plane of the horizon. But the sides joining G to the extremities of the perpendicular include the colatitude and, therefore, angle LFO equals the colatitude of the city.

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If the feet of the vertical lines, dropped from the tips of the three rods upon the plane of the horizon are collinear, then our place of observation must be on the equator. That is so because the sines of the altitudes, for any one parallel, project at the equator into one straight line. The plane of the parallel and that of the circle of altitude are both perpendicular to the plane of (MS 55) the horizon at the equator, therefore their common line of intersection is perpendicular to the plane of the horizon, and hence it is the sine of the altitude. Now, because the sines of the

altitudes are in the plane of the parallel, and the plane of the horizon intersects it in a straight line, hence they fall along it.

Further, in all other localities, since the sines of the parallel fall perpendicularly on the horizon plane, and since the plane of the parallel is inclined to it, therefore the locus of the projections of the sines is the periphery of an ellipse which is the intersection between the horizon and the oblique cylinder whose sides are those sines.

These three rods can also be used for the sun;

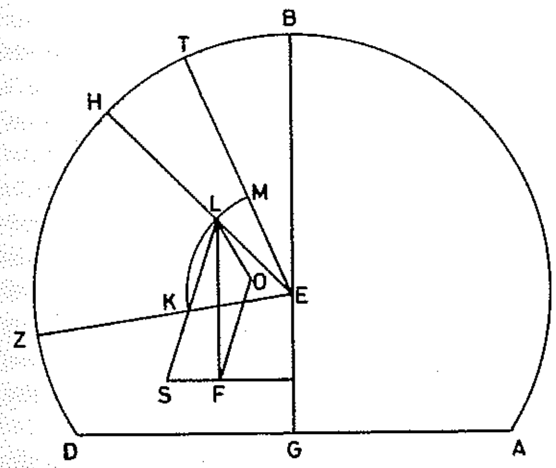

Figure 3

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its observation would be easier because of its intense light and the facility of obtaining collinearity of the rods with the straight lines, either by their own shadows or by the emergence of the ray from the apertures of the two visors. It is also possible to determine the latitude of a city by making the following observation: We construct on the horizon, with extreme care and precision, a large hemisphere, and we determine on it the point which is vertically below the zenith. This point is in the midst of

its surface, and is such that a plumb line situated at it would be equally inclined to the surface. When that is done, we make a circle, like the hoop of a tambourine, of diameter about a span, and we build up on top of it, as a base, a right (MS 56) circular cone. We conjoin the base and the cone with a lattice, so that it is possible to look through the lattice into the interior of the cone, and to pick up with the hand, through the base, any object inside the cone. Then we pierce, inwards, a fine hole through the apex of the cone; we prop up the base with light wood that touches the surface of the sphere but does not obstruct sliding on it, and we determine the center of the base on the wood. Then we observe the sun by putting the base of the cone on the surface of the hemisphere and sliding the base slowly, meanwhile looking through the lattice until the sun's ray through the apex of the cone falls on the center of its base. When this takes place, we mark the point on the surface of the sphere immediately under the center of the base. We wait for a part of the day, and then repeat the observation as described, for a second time, and later for a third time. We then consider the three points which we have marked on a single day, and seek, on the surface of the hemisphere the pole of the circle that passes through those points. The pole which we have sought is in the direction of the north pole, and the great circle arc between it and the zenith is the colatitude of the city.

A complete sphere of perfect construction may also be taken, and may be placed on any plane, whether parallel to the horizon or not, but the sphere should be

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firmly held in contact with the plane to prevent any motion, or change of position. Then a gnomon is constructed on a plane base of some width, that can elegantly touch the surface of the sphere, (MS 57) and is adjusted to stand at equal angles to its base (i.e. the gnomon stands at the center of a circular base and is normal to it). A position on the sphere, exposed to the sun, is then sought where, if the gnomon is put in that position, its shadow would be nil. When this position is found a circle is drawn round the base of the gnomon, and this observation is repeated three times on the

same day. Then the centers of the three circles of the base are marked, and the pole of the circle through the three centers is sought on the sphere and determined. This pole is in the direction of the north pole. Further, a position on the sphere is sought such that, if the gnomon is put in that position, and a plumb line with a sharp plumb bob is suspended on top of it, the head of the gnomon would go down, and if the gnomon rod is removed from its base, the sharp head of the plumb bob would fall into the center of the base. In that position the center of the base will be in the direction of the zenith, and the great circle arc between it and the first point is the colatitude of the city. If this is deducted from ninety, there remains the latitude of the city. Both methods are based on the same principle, but once the sphere is made and set up, the latter method is easier to adopt, and less costly.

It is also possible to determine the latitude of a city by a method which is simpler than these, namely, by observing at two different times, two altitudes of the sun, or a star, and the azimuth corresponding to each altitude.

Let circle ABG (Fig. 4) represent the horizon, AEG the meridian line, BE the east-west line, ZD the line of intersection between the planes of the horizon and the sun's daily path. Also, let BM be the measure of the azimuth of the first (MS 58) altitude, from the east-west line, and BH the measure of the azimuth of the second altitude. We draw ME and HE, and we drop MS and HX perpendicular to BE. Further, let EO represent the cosine of

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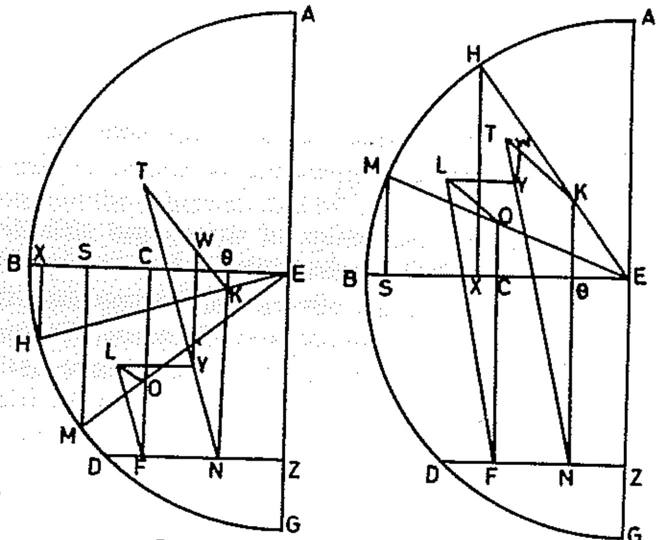
the first altitude, EK the cosine of the second altitude. Also, let us drop OC (and) KΘ perpendicular to BE, and let OL (and) KT be perpendiculars to the plane of the horizon such that OL equals the sine of the first altitude, and KT equals the sine of the second altitude. We draw LF and TN. The triangle LOF is the triangle of the first altitude ; the triangle TKN is the triangle of the second altitude, and the two triangles are similar, because all such triangles of the daily path are similar.

We construct the lines LY and YW parallel to the horizon. (LY is constructed parallel to DZ and YW parallel to GE) The triangle TWY is also similar to the two triangles of altitude.

Because the triangles EMS (and) EOC are similar, the ratio of EO, the cosine of the first altitude, to OC, the share of the first azimuth, is as the ratio of EM, the total sine, to MS, the sine of the first azimuth. So OC can be determined, and in like manner the ratio of EK, the cosine of the second altitude, to KΘ, the share of the second azimuth, is as the ratio of EH, the total sine, to HX, the sine of the second azimuth. So KΘ can be determined. The difference between OC (and) KΘ, which equals WY, is determinable, and likewise the difference between OL (and) KT, the sines of the two altitudes, is determinable, (MS 59) it being WT. So TY, the hypotenuse of a right triangle of which the known TW (and) WY are legs, is (also) determinable. And the ratio of TY to TW is as the ratio of the sine of angle TWY, the right angle, to the sine of angle TYW. But angle TYW is in amount the colatitude of the city. So it is determined, and (hence) the latitude of the city is determined.

This operation is divisible into five parts: First, when both azimuths are north of the east-west line. Second, when both are south. Third,



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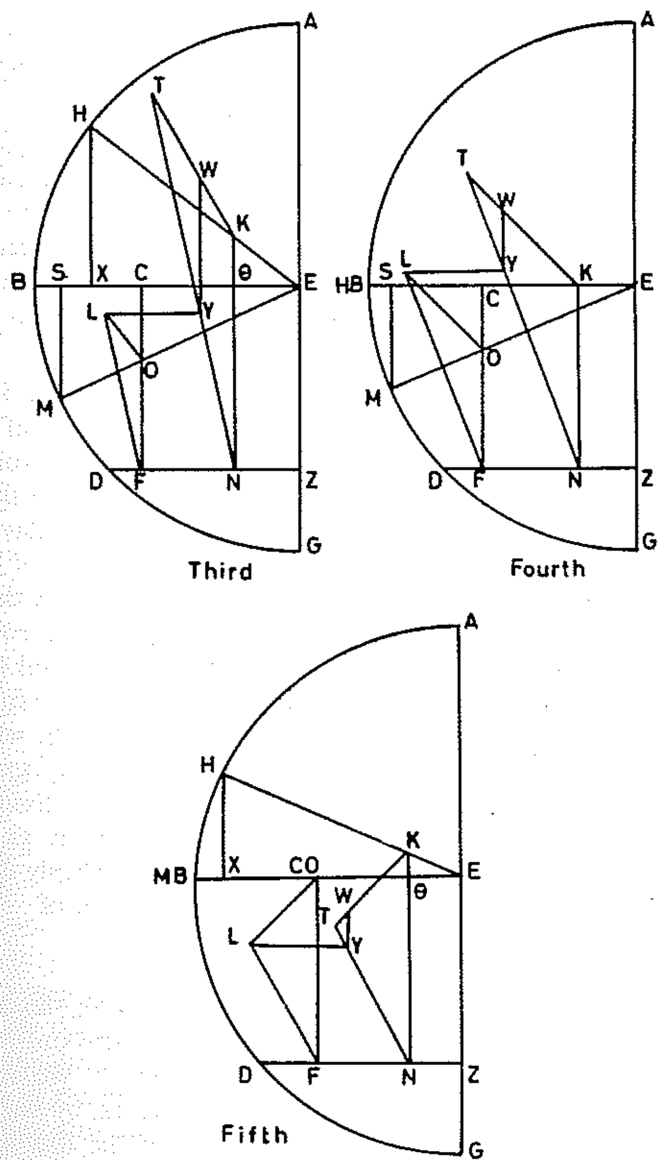

Figure 4

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when one is north and the other is south. Fourth, when one is north and the other is on the east-west line. And fifth, when one is south and the other is on the east-west line.

The first, third, fourth, and fifth cases deal with parallels of northern declinations. The second case deals with parallels of northern, southern, or zero declination. Hence this case requires three figures, but, for brevity, we have presented only on figure, because the following numerical example can serve as a substitute for one of the figures of this case. (MS 60 shows Fig. 4, then MS 61 follows).

For brevity, I shall condense the computation in this example, and so I say: For astronomical purposes, one of which was the determination of latitude, I observed the sun at Jurjāniya twice in the afternoon of Friday, the fourth of Rajab, in the year four hundred and seven of the Hijra, (being) Ashtadh day, 26 of the month Adhar, in (year) three hundred eighty-five of Yazdigerd. The solar altitudes and azimuths at two times on that day are: As for the first (observation), the altitude was 21;10°, and the azimuth 67;30° from the west point. At the second (observation) the altitude was 14;50°, and the azimuth 52;30° from the west point. We multiply the sine of the first azimuth, which is 55;25,58 by the cosine of the first altitude, which is 55;57,7, and we get the product 40,196,369,266 fourths. We divide this product by the total sine, and we get the quotient 51;41,35, which is the share of the first azimuth. We also multiply the sine of the second azimuth, which is 47;36,4, by the cosine of

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the second altitude, which is 58;0,1, and we get 35,780,974,564 fourths. We divide this product by the total sine, and the quotient is 46;0,53, which is the share of the second azimuth. We multiply the difference between the shares which is 5;40,42, by itself, and we get 417,875,364 fourths. Now, the sine of the first altitude is 21;39,54 (MS 62) and the sine of the second altitude is 15;21,38. Therefore the difference between them is 6;18,16, and the square of this difference is 515,108,416 fourths. The sum of the two

squares in fourths is 932,983,780, and its (square) root is 30,545 seconds, which is the measure of the hypotenuse. We multiply the difference between the sines of the altitudes by the total sine and we get 1,361,760 seconds. We divide this product by the seconds of the hypotenuse, and we get the quotient 44;34,55 which is the cosine of the latitude of the city. Its arc (sine) is 47;59,25°, and therefore the latitude of Jurjāniya is 42;0,35°.

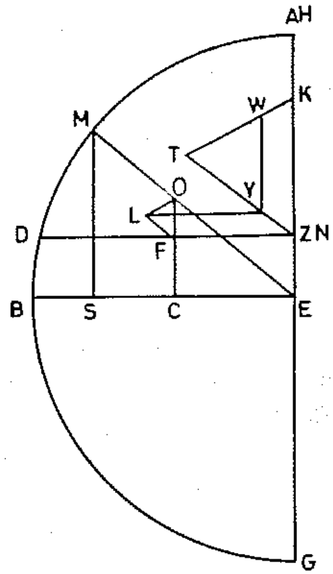

Figure 5

If one of the altitudes is a meridian altitude, it being necessarily the greater, then AE (Fig. 5) which is a segment of the meridian line, would take the place of EH; EK would be the cosine of the meridian altitude, and KT would be the sine of the meridian altitude. The rest of the demonstration would proceed as explained above. But the problem is divisible into five cases, because the meridian altitude may be south of the zenith, through the zenith, or north of it, and the azimuth of the other altitude

in the first case may be either on the east-west line, or displaced from it to the south, or to the north, but in the other two cases it can only be displaced to the north. In all the examples provided, I am excluding places on (MS 63) the equator and those whose latitudes are not less than the complement of the sun's maximum declination, for we have no reliable reports that such places are inhabited. I shall limit the illustration of these cases to a consideration of the following observation, because the other cases can be dealt with similarly.

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On that Friday whose date was mentioned above, I observed at noon the sun's altitude at Jurjāniya and found it to be 24;28°. KT, the sine of this altitude, is 24;50,59, and KE, its cosine, is 54;36,44. This observation was followed by two more, one after the other. If we suppose that BM is the azimuth of the second altitude, then BM is 67;30°, and MS, its sine, is 55;25,58. The altitude of this azimuth is 21;10°. LO, its sine, is 21;39,53. OE, its cosine, is 55;57,7. If we multiply EO by MS, the product is 40,196,369,266 fourths, and if we divide this product by the total (MS 64) sine the quotient is 51;41,35, which is the share of the azimuth, I mean OC. The difference between it and KE, the cosine of the meridian altitude, is 2;55,9, which is YW. Now the difference between LO and TK, 3;11,6, is TW. Hence the square of TW is 131,469,156 fourths, the square of YW is

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110,439,081 fourths, the sum of the two squares is 241,908,237 fourths, and its (square) root is 15,553 seconds of the chord. We multiply the difference between LO and TK by the total sine, and get the product, 687,960 seconds. We divide this product by the seconds of the hypotenuse and get the quotient 44;13,59, which is the cosine of the latitude of the city. The arc (sine) of this latitude is 47;29,42°, and therefore the latitude of Jurjāniya is 42;30,18°.

If we suppose that arc BM is the azimuth of the third altitude, measured around midday, then BM is 52;30°, and MS, its sine, is 47;36,4. Now, the altitude of this azimuth is 14;50°.

LO, its sine, is 15;21,38. OE, its cosine, is 58;0,1, and OC, the share of the azimuth is 46;0,53, and WY, the difference between OC and KE, is 8;35,51. Its square is 957,964,401 fourths. TW, the difference between LO and TK, is 9;29,21. Its square is 1,166,290,801 fourths. The sum of the two squares is 2,124,255,202 fourths, and the (square) root of this sum is 46,090 seconds for the hypotenuse. The product of (MS 65) TW by the total sine is 2,049,660 seconds, and if we divide this by the second parts of the hypotenuse, we get the quotient 44;28,15, which is the cosine of the latitude of the city. Its arc (sine) is 47;49,56°, and therefore the latitude of Jurjāniya is 42;10,4°.

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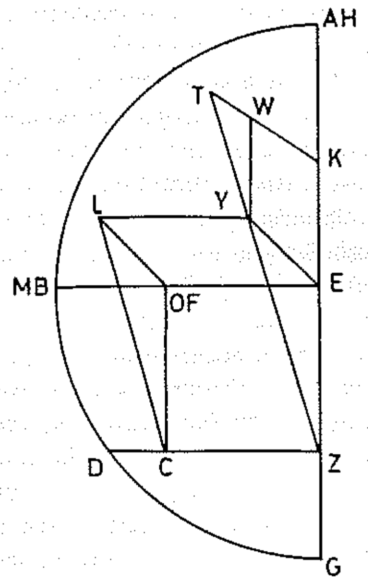

Figure 6

If one of the two altitudes is a meridian altitude and the azimuth of the other altitude is on the east-west line, the figure corresponding to this case is as shown (Fig.6).

I made two observations, one of them in a village called Būshkānz, west of Jayḥūn, between Jurjāniya and the city of Khwārizm. It was made in the year three hundred eighty-four of the Hijra, that (being) the year three hundred sixty-three of Yazdigerd, - by means of a (MS 66) horizontal circle of diameter fifteen cubits. I measured its (the sun's) maximum altitude by its shortest shadow during the year and found it to be 71;58,45°. I also obtained the length of the shadow when it lay along the east-west line on that day, but I have forgotten its amount, because of confused conditions at the time, which made it necessary to evacuate and interrupt the work. However, I do remember that I was able to obtain the maximum declination: 23;35,45°, and the latitude of the village: 41;36°.

The second (observation) was made in the year four hundred seven of the Hijra, when I observed, at Jurjāniya, the maximum meridian altitude with a quadrant of a circle of diameter six cubits, whose circumference was graduated in minutes, and found that altitude to be 71;18°. My mind was not at ease about the real minimum altitude; therefore, as precautionary measure, I observed the altitude whose azimuth is zero, on the middle day of the interval of days,

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when the altitudes were approximately equal empirically. That altitude was observed on Friday, the seventh of Muḥarram of the year mentioned above, on Khurdad day, the 6th of the month Tir, of the year three hundred eighty-five of Yazdigerd, and I found it to be a little less than 36;30°. Its sine is 35;41,22, which is the measure of line LO; the sine of the meridian altitude is 56;49,57, which is the measure of TK. The difference between them is 21;8,35, which is equal to TW, and its square is (MS 67) 5,793,493,225 fourths. But YW is equal to EK, which is the cosine of meridian altitude, 19;14,12. Its square is 4,795,839,504 fourths. The sum of the two squares is 10,589,332,729 fourths; the (square) root of this sum is 102,904, which is the measure of the hypotenuse in seconds. But the ratio of TY to YW is as the ratio of the sine of angle TWY, a right angle, to the sine of angle WTY which is the amount of the city's latitude. Therefore,

if we multiply the cosine of the meridian altitude by the total sine, we get 4,155,120 seconds, and if we divide this by the seconds of the hypotenuse, we get the quotient 40;22,43, which is the sine of the latitude of the city. Its arc (sine) is 42;17,50°, which is the latitude of Jurjāniya.

The same thing comes out in different amounts, because celestial observation is a very delicate matter; it requires precise measurements of parts of big celestial orbs by minute parts of a small circle in the instruments, and hence the complete identification of parts can only be an approximation. The approximation is also due to the extraction of square roots in the calculation of chords and sines, and the lack of refined methods for the calculation of some quantities like the chord of one part out

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of 360 parts of a complete rotation, and hence very minute defects, in operations including sines, lead to defective (MS 68) composite approximations. Similarly, minute defects of observation influence slightly what is derived by computation. But I do not use it (derivation by computation), except for exploring the truth behind the veils, and for comparing results arrived at by different methods, in order to feel more confident about a derived result. An accurate measure of the latitude of Jurjāniya is 42;17° because, if we deduct from the altitude of the summer solstice 71;18°, which we observed and found, the maximum declination, 23;35°, there remains 47;43°, which is the colatitude of the city. Hence the latitude itself is 42;17°; it is my reliable estimate which I use in my work. It is all the same whether we do that or add the maximum declination to the complement of the maximum altitude which is 18;42°, for if we add this to the maximum declination, the sum is also 42;17°, the latitude of Jurjāniya.

If the observer is situated on the equator, where the sines of the altitudes are in the plane of the parallel, the line segment LO coincides with the line segment LF. Similarly, the line segment TK coincides with TN. Therefore the two triangles OLF and KTN cease to exist. As each side of the pair LF and TN includes (respectively) with each side of the pair, EC and NΘ, an angle equal to the colatitude of the city (i.e. angle LFC =

angle TNΘ = colatitude), those two pairs would, also there, include the colatitude. That is so because they (LF and TN) are (respectively) perpendicular to FC and NΘ. And since the complement of the latitude there is a whole quadrant, hence the figure (Fig. 7) is as shown (MS 69) for the case in which equality holds between KΘ and OC, the two shares of the two azimuths. Whenever they come out equal, it follows that the locality is on the equator.

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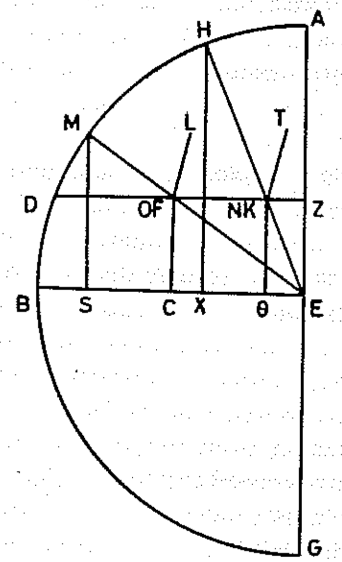

Figure 7

It is obvious that if one of the altitudes is a meridian altitude, EZ is equal to the share of the azimuth, and is equal to the cosine of the meridian altitude, or the sine of the rising amplitude. But DB, the amplitude of the rising point, is equal there to the maximum declination, so whenever the cosine of the meridian altitude is found equal to the share of the azimuth of the other altitude, the place of observation is on the equator, and the

share of the azimuth there is always equal to the sine of the amplitude of the rising point, I mean the sine of the declination. If the sun is at one of the two equinoctial points, the projections of the sines of the altitudes fall on line BE, and the observer of any altitude will find no azimuth except the east-west line.

It is also possible to derive the unknown latitude of a city from the known latitude of another if observations are made in both cities of one and the same fixed star, either

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simultaneously or during an interval between two times, while the fixed stars do not seem to move. If, while the star is on the meridian circle, its altitudes in both cities are obtained, and if the star is on the same side (MS 70) of the zenith in each of them, we take the difference between its altitudes at both cities. Now, if both altitudes are south of the zenith, and if its altitude at the city of known latitude is the bigger, we add that difference to the known latitude, but if its altitude there is the lesser, we deduct that difference from the known latitude. Further, if both altitudes are north of the zenith, and if both are in upper culmination, or both are in lower culmination, in the case of a circumpolar star, we add the difference of altitudes to the known latitude, if its altitude there is the lesser, but we deduct the difference, if its altitude there is the bigger. Also, if one altitude is north of the zenith at one city, but south of it at the other, we add the two coaltitudes and increase the known latitude by that sum, if the star's altitude there is north of the zenith, but we deduct that sum if its altitude there is south of the zenith. The result of the operation outlined, in all cases, is the unknown latitude of the other city.

Proof: Let ABGD (Fig. 8) be the meridian circle, BED the meridian line of the city Z, AEG the meridian line of city H. Let us suppose that K is the transit position of a star, south of both H and Z, and that O is an intersection of the equator with the circle (MS 71) ABGD. If Z is the city of known latitude, we take the difference of the star's altitudes, they being KB and KA, which is AB and equal to ZH, and if we add this to ZO, the latitude of the locality, Z, the sum is OH, which is the

latitude of city H, because KB, the altitude at Z, is bigger than KA, the altitude at H. Now, suppose that H is the city of known

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latitude, then KA, the altitude there, is less than KB, the altitude at city Z. Therefore, if we deduct the difference HZ from the latitude OH, there remains OZ, the latitude of city Z. The same demonstration holds if we suppose that the star transits through the zenith at city Z, for if the known latitude is OZ, and if we add this to the difference, we get the sum OH. That is so because the latitude at city Z is the bigger, but if the known latitude is OH, then, if we deduct from it the difference of altitudes, there remains OZ.

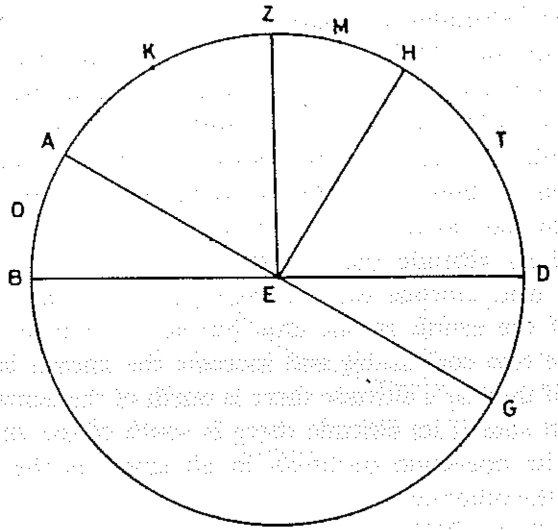

Figure 8

If we suppose that the star transits at a point T, north of the zenith at both cities, Z and H, then the condition of increase and that of decrease are reversed, for if OZ is the known latitude and TD, the altitude at Z, is less than TG, which is the

altitude at H of unknown latitude, we add the difference HZ to OZ, and the sum is OH, which is the latitude of city H. But if the known latitude is OH and the altitude TG at H is bigger than the altitude TD at Z, the city of unknown latitude, then we deduct the difference from the latitude OH, and thereby obtain OZ, which is the latitude of (MS 72) city Z.

The previous demonstration holds if the star transits through the zenith at H. However, if we suppose that it transits at M, between Z and H, north of Z but south of H, then HM is the complement of the altitude MA at city H, and ZM is the complement of the altitude MD at city Z, hence the sum of the complements is HZ. If OZ is the known latitude and M is north of Z, we add the sum HZ to latitude OZ and the sum is OH, but if OH is the known latitude, and the star M is

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south of H, then we subtract the sum HZ from the latitude OH, and there remains OZ.

I have not found anything in the recorded observations which can be used for illustrative purposes except the star adjacent to **As-Suha,** I mean: the middle one of the three **Banāt-Naᶜsh.** The sons of Mūsa (bin Shākir) found that its altitude at upper culmination is 63;5° at Surra-man-ra'ā (Samarra). They also remarked, as stated previously, that its altitude at Baghdad is 62;13°, and hence the difference (MS 73) between the altitudes is 0;52°. Since the star is north of the zenith at both cities, and the latitude of Surra-man-ra'ā, according to other observations is 34;12°, if we deduct that difference from this latitude, there remains 33;20°, which is the latitude of Baghdad, according to their observations, and if we add that difference to it, (the latitude of Baghdad) we recover the latitude of Surra-man-ra'ā. I said before that the altitude of this star was found to be 62;3° in

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some copies. Hence the difference of altitudes is 1;2°, and if we deduct this amount from the latitude of Surra-man-ra'ā, there remains 33;10°, the latitude of Baghdad. This shows that the first (altitude) is more accurate, and that the difference is due

to an error in copying.

As it was possible to apply this method to the fixed stars, it is also possible to apply it to the sun, but during a day of a fixed date, because the declination, which varies during some hours, affects the amounts of the altitude. We can illustrate the application of the method to the sun by quoting the following reports. I have found in the Damascene records that the sun's noon altitude at Damascus was 72;7,50° on Wednesday, twenty-sixth of Rabīᶜ al-Awwal, year two hundred seventeen of the Hijra, Isfandarmadh day, the 5th of the month of Farvardin, year two hundred one of Yazdigerd. Abū al-Ḥasan recorded that he found it 72;14° at Baghdad. Hence the difference between them is (MS 74) 0;6,10°, and if we deduct this from the latitude of Damascus, which was found to be 33;30,18°, there remains 33;24,8°, which is the latitude of Baghdad.

It was also reported that the noon altitude at Damascus was found to be 73;2,4°, on Saturday, the second of Rajab, year two hundred seventeen of the Hijra - Adhar day, the 9th of the month Tīr, year two hundred one of Yazdigerd. Abū al-Ḥasan recorded that he found it in Baghdad to be 73;7°. The difference between them is 0;4,56°. If we deduct this from the latitude of Damascus, there remains for the latitude of Baghdad: 33;25,22°.

Also, another example: Abū Maḥmūd al-Khujandī found in the year

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three hundred eighty-four of the Hijra, at Rayy (Rhagae), that the sun's maximum altitude was 77;57,40°, and that the latitude of Rayy is 35;34,39°. I found in that year that the sun's maximum altitude, at one of the villages of Khwārizm, was 71;59,45°, and that the latitude of the village is 41;36°. The difference between the two altitudes is 5;57,55°; if we add this difference to the available latitude of Rayy, we get 41;32,34°, which is the latitude of the village. But if we deduct the difference from the available latitude of the village, there remains 35;38,5°, which is the latitude of Rayy. I give different kinds of examples for the same principle, because I wish to emphasize the evidence, and to give more confidence with the corroboration of results.

If the two altitudes of one and the same fixed star are observed simultaneously, or at two (MS 75) nearby times, the problem (of latitude) is dealt with by the procedure discussed before, but if the two times are far apart, or if the altitude is that of an upper culmination at one place and a lower culmination at the other, then a knowledge of the latitude and longitude of the star is indispensable, and therefore I have left this problem to (be solved by) a zīj, which is the proper place for it.

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III. ON THE INDEPENDENT DETERMINATION OF THE MAXIMUM DECLINATION (THE OBLIQUITY OF THE ECLIPTIC)

The maximum declination is the magnitude of the angle formed by the intersection of the plane of the celestial equator and the plane of the ecliptic. It is also called the total declination, and is equal to the arc between the poles of those two circles. Its determination, without the aid of the latitude of the place of observation, is made by two methods: (MS 76) The first method is to obtain the maximum altitude of the sun in the celestial meridian at the place of observation, and also its minimum altitude at that place. The difference between these altitudes, when they fall on the same side of the zenith, or the sum of their complements when they fall on opposite sides of it, is equal to twice the maximum declination. The second method is to obtain one of those two altitudes, and also an altitude of the sun, of known azimuth, on that day.

The first method is more reliable, because it depends on direct observation and does not involve any computation. It was used by the ancients, as well as most of the moderns. The works of some of them, like those of Eratosthenes, have not been transmitted to us. However, according to the Almagest, Hipparchus relates (as follows:) Eratosthenes held that the arc between the two solstices is approximately eleven parts out of eighty-three parts of a complete rotation, and he agrees with him, accepting it (this ratio). It is not known whether this was handed down by tradition, or was the result of an actual observation. However, there is an obvious simplification in this amount, for it is the practice of astronomers to divide circles - in particular great ones - into three hundred and sixty parts, and the arcs of their instruments are graduated on this basis. So the number quoted above was not obtained from an actual graduation of a rotation;

the actual number was transformed into it, either to round off the fractions, or for another purpose which is better known to its author.

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The ratio of eleven to eighty-three is equal to the ratio of the arc between the two solstices to three hundred and sixty, where a complete rotation is made of three hundred and sixty parts. If the first (proportional) is multiplied by the fourth, and if the product is divided by the second, the third comes out with vacant eighth parts, but not discontinued thereafter; it is 47;42,39,2,10,7,14,13 (MS 77) and half of this is 23;51,19,31,5,3;37,6,30, with additional parts of lower orders. It is known that, as far as human ingenuity can perform, the instrument can be graduated to read third parts only, and even this is hardly accurate. So there is no doubt that the amount of this arc, registered by the instrument, is not those given parts, because they cannot be eaxctly transformed into those two numbers, and because they cannot be realized empirically. The approximation which was mentioned with the parts testifies to what I have said.

In the first treatise of the Almagest, Ptolemy stated that, for several years in succession, he observed with a ring, which was erected in the circle of the meridian on a shaft that revolved inside it, and which had, in its plane, another ring that was provided with two diametrically opposite visors; and that he observed also with a quadrant of a circle, made on a plinth (libna) erected in the plane of the circle of the meridian, whose center was at the foot of the gnomon that was erected at its upper southern corner. At all times, he found it (the arc between the two solstices) to be forty-seven degrees, and more than two thirds, but less than three quarters of a degree. He assumed that this amounts approximately to what Eratosthenes had said, which was accepted by Hipparchus. He said that because the rule is - for such a range with an upper limit and a lower one - to take the average amount between them. Hence the amount (MS 78) given by Ptolemy is 47;42,30°; its half is 23;51,15°, but he constructed the tables of declination on the basis of 23;51,20°, in agreement with that assumed by Hipparchus and Eratosthenes,

for if their third parts are rounded off, the declination comes to be this amount.

After Ptolemy, we have no record of any astronomer's observation, up to the time of al-Ma'mūn, the Commander of

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the Faithful, who ordered Yaḥyā bin abī Manṣūr to make a new determination. He (Yaḥyā) made it at Shammāsiya, and it is well known that he found the maximum declination to be one hundred and fifty-seven parts out of two thousand and four hundred parts of a complete rotation. This gives 23;33°, and on this basis he (Yaḥyā) constructed his zīj. That was reported by al-Khwārizmī who was an eye-witness to the observation. They found that the maximum altitude is 79;6° and that the minimum altitude is 32;0°. Therefore the difference between them is 47;6°, and half this amount is 23;33°. That was in the year two hundred thirteen of the Hijra, which is the year one hundred ninety-seven of Yazdigerd. Yaḥyā bin abī Manṣūr passed away before al-Ma'mūn launched his campaign against the Byzantines.

When it was found at Shammāsiya, in the year two hundred fourteen of the Hijra, (being) one hundred ninety-eight of Yazdigerd, that the maximum altitude was 80;8°, and that the minimum altitude was 32;58°, then by the rule of (MS 79) half the difference between them, the declination was found to be two hundred and eighty-three parts out of four thousand three hundred and twenty parts of a rotation, i.e. 23;36°. Al-Ma'mūn rejected the first observation, and said that it was not valid, not for the difference in the amount of the declination, but for the big difference between the two altitudes. Again, al-Ma'mūn ordered Khālid bin ᶜAbd al-Malik al-Marwarūdhī to observe at Damascus. He constructed a huge mural quadrant (**libna**) of side ten cubits on the mountain where the convent of Murrān (Dayr Murrān) is situated, and set up on the circumference of

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the marble quadrant a dirigible brass instrument which was provided with an aperture through which the sun and the peg at the center of the quadrant could be seen. He observed with it

daily for a full year, part of which fell in the year two hundred sixteen and the other part in the year two hundred seventeen of the Hijra.

But, concerning the declination, it was reported that he found the minimum altitude in year two hundred sixteen (to be) 32;56°, the maximum (altitude) in year two hundred seventeen (to be) 80;3,55°, and the minimum altitude in year two hundred eighteen (to be) 32;55°. This third is not reliable, because the period of observation was only approximately for one year. Now, if we measure the first against the second the declination would be 23;33,57,30°; but if we relate the second to the third, it would be 23;34,27,30°. This should be disregarded for the reason which I have stated, and because Sanad bin ᶜAlī, who supervised the work of Khālid, has reported that he (Khālid) found the declination to be 23;33,52°, which is identical with what comes out by measuring the first to the second. It is possible that these (reported) seconds were originally fifty-seven, as we have found them here, but that they have been erroneously copied. Also, tables have fallen into my hands, which give the results of Khālid's observations of the sun's altitudes on the meridian circle at Damascus, and they show that the two solstices did not fall at midday. That is so because at midday on Monday, the tenth of Jumādā I, year two hundred seventeen of the Hijra, Bādh day, 22 of the month of Urdībihisht, year two hundred one of Yazdigerd, the greatest altitude at Damascus (was found to be) 80;4,30°, and on the previous Sunday (it was found to be) 80;4,10°, and on the following Tuesday, 80;4,28°.

Let AB and BG (Fig.9) be two equal arcs of the ecliptic, and let A be the point whose altitude was found on Sunday, B be the point whose

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altitude was found on Monday, and G be the point whose altitude was found on Tuesday. It is known that the altitude of B is the greatest of the three; it is either the solstitial point, or nearer to it than either (MS 81) A or G. Now, if it were the solstitial point, the altitude of A would have been equal to the altitude of G, because A and G are on both sides of it and at equal

distances empirically. But they were not found equal, therefore B is not the solstitial point, and since it is the summer solstice, the point nearer to it has a greater altitude than the point farther from it. But the altitude of G is greater than that of A, therefore point G is nearer the solstice than point A. Similarly, B is nearer to it than A, and therefore it is between B and G. Let it be represented by point E.

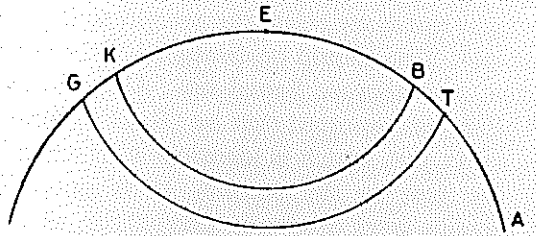

Figure 9

About E as pole, and with radii EB (and) EG, we draw the small circles BK and GT. It is obvious that the declination at point T is equal to that at G, and hence their noon altitudes are equal. According to what is done in most of the operations of zījes, even though it is an approximation and simplification, let the ratio of the difference between the altitudes of T and A, it being 0;0,18°, to the difference between the altitudes at B an A, it being 0;0,20°, be as the ratio of AT to AB. Now AB is the apparent travel of the sun between the two noons, (that of) Sunday and (that of) Monday. The distance of this arc, at the time of the observation, from the apogee is eight degrees. Hence it (AB) is 0;56,58,48°, and hence arc AT is 0;51,16,55. But we took AB and BG as equal, and TB and KG are equal. So the arcs AT and BK are equal. Hence the sum of half BK and AB, which is AE, must be 1;22,37,15,30°. And the ratio of AB to AE is as the ratio of the difference between the altitudes at A and

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B to that between the altitudes at A and E. So the difference between the altitudes at A and E therefore is 0;0,29°. Hence, if we add it to the altitude of A for Sunday, the result is 80;4,39°, which is the maximum altitude of the sun at Damascus. Now we discuss the minimum altitude there. On Tuesday, the twentieth of Dhū al-Qaᶜda, year two hundred seventeen of the Hijra, Dīn day, 25 of the month Ābān, year two hundred one of the Persians, the noon altitude given in these tables is 32;54,58°; that at noon on the previous Monday is 32;55,0°, and that at noon on the succeeding Wednesday is 32;55,28°.

Let A, on the ecliptic, represent the point of Monday, B the point of Tuesday, and G the point of Wednesday. Then, as in the previous demonstration, (MS 83) E, the solstitial point, must lie between A and B (Fig.10). So the ratio of the difference between the altitudes of points G and T, which is 0;0,28°, to the difference between the altitudes of points G and B, which is 0;0,30°, is as the ratio of GT to GB. But GB is at a distance of nine degrees from the point opposite the apogee at the time of the observation. Hence GB, which is the true travel of the sun (during the time) between the noons of Tuesday and Wednesday, is 1;1,27,36°. Therefore GT will be 0;57,21,46°. And because of the equality of TG and KB, if we add BG to half of

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TG we get GE. Hence it (GE) is 1;30,8,29°. The ratio of GB to GE is as the ratio of the difference between the altitudes of G and B, which is 0;0,30°, to the difference between the altitudes of G and E. So the difference between the altitudes of G and E is 0;0,44°. But the altitude of E is the least altitude, therefore, if we deduct that difference from the altitude of G, there remains 32;54,44°, which is the altitude of the winter solstice at Damascus. So the maximum declination according to these two (MS 84) altitudes is 23;34,57,30°.

People do not carry out their evaluations to such a degree of precision. If they derive the greatest declination from those

Similar observations are mentioned in a work by al-Nayrizi, Ms. Leiden, Warn. 556, (... commentary on the first book of the Almagest)

tables, they would get 23;34,51°, because the greatest (altitude) in them is 80;4,30°, the least (altitude) in them is 32;54,48°, and half the difference between them is the greatest declination.

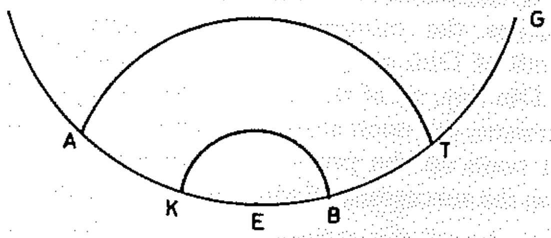

Figure 10

Further, Muḥammad and Aḥmad, the sons of Mūsa bin Shākir, observed the maximum altitude at Surra-man-ra'ā, and found it to be 79;22°, at noon on Thursday, the twentieth of Ṣafar, in the year two hundred forty-three of the Hijra. They found that the minimum altitude there was 32;13° at noon on Thursday, the twenty-fifth of Shaᶜbān, in the year two hundred forty-three of the Hijra, which is Anīrān day, 30 of the month Ābān, year two hundred twenty-six

June 857

Dec 857

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of Yazdigerd. They also observed this minimum altitude, 32;13°, at noon on Sunday, the seventeenth of Ramaḍān, year two hundred forty-five of the Hijra, Ahnūdh day, the first of the epagomenal days, year two hundred twenty-eight of Yazdigerd. The difference between the two altitudes is 47;9°. Half of it, which is 23;34,30°, is the greatest declination.

Dec 859

Later on, they observed at Baghdad, from their house on the bridge, according to reports by Abū al-ᶜAbbās al-Nairīzī, and Abū Jaᶜfar al-Khāzin, in their commentaries on the first treatise of the Almagest. They found that on Thursday, (MS 85) the twenty seventh of Dhū al-Ḥijja, year two hundred fifty-four

Dec 868

Nayrizi ...

of the Hijra, Isfandmādh day, the third of the epagomenal days, year two hundred thirty-seven of Yazdigerd, by (the use of) two rings together, that the minimum altitude was 33;5°, and on Friday, the fourth of Rajab, year two hundred fifty-five of the Hijra, Hurmuzd day, the first of the month Khurdādh, year two hundred thirty-eight of Yazdigerd, that the maximum altitude was 80;15°. The difference between them is 47;10°, and its half, which is 23;35°, is the maximum declination.

Jun 869

After these, Muḥammad bin Jābir al-Ḥarrānī, known as al-Battānī, observed the maximum altitude, at the city of Raqqa, with the usual mural quadrant (**libna**) but he provided it with an alidade. He found that the sun's nearest zenith distance was 12;26°, and that its farthest distance from the zenith was 59;36°. The difference between them is 47;10°, and hence the maximum declination is 23;35°. He declared that he had repeated his observations over many years, and found the declination (to be) as we have mentioned, but he did not give specific dates. We know, however, that his observations were made between the years one thousand one hundred ninety-one and ninety-four

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of Alexander, that being between the years two hundred sixty-seven and seventy of the Hijra.

Later on, Sulaimān bin ᶜIṣmat al-Samarqandī observed, at the city of Balkh, with a mural quadrant (**libna**) provided with an alidade, of diameter eight cubits, and found that its minimum altitude there was 29;46°. (MS 86) The solstice, however, was not truly on the meridian, so he transformed it (the altitude) into it (the meridian) and it became 29;44,44°; then he corrected it for parallax and it became 29;47,17,6°. This observation was made on Saturday, the seventh of Shaᶜbān, the year two hundred seventy-five of the Hijra, Hurmuzd day, the first of the month Adhār, year two hundred fifty-seven of Yazdigerd.

He also observed the maximum altitude on Tuesday, the fourteenth of Muḥarram, year two hundred seventy-six of the Hijra, Khurdādh day, the sixth of Khurdādh month, year two hundred fifty-eight of Yazdigerd, and found it to be 76;54°. The solstice, however, was after the passage of the meridian; so he

transformed the altitude to it and it became 76;54,4°, then he corrected it for parallax and it became 76;54,41,23°. If we adopt the geometrical rule about the meridian altitudes, the declination would be 23;34°, but if we use the altitudes of the solstices, it would be 23;33,42,8,30°.

It has been related in some anecdotes that Manṣūr bin Ṭalḥa made observations for the declination and found it to be 23;30°, and in other stories that he found it to be 23;34,44,30°, but such stories cannot be fully trusted. This learned gentleman was the last of the Ṭāhirid governors

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in Khurāsān, and he had some knowledge of mathematics and related sciences. The observations of Sulaimān bin ᶜIṣmat were made (MS 87) in his days, so it is quite possible that he used the declination which was found by Yaḥyā bin Abī Manṣūr, and also that which was found later by Sulaimān, and pleaded before his audience that they were based on actual observation, leaving the hearer under the impression that he himself had made the observations. He actually worked on the determination and correction of the longitudes and latitudes of the cities of Khurāsān, and this made him a central figure in stories. However, he would be credited with observation of the declination if there had been more detailed reports about it.

In the second chapter of Manṣūr's book, **Fī al-ibāna ᶜan al-falak,** we read: «The (maximum) declination is 23;34° and some seconds, according to our investigations». One is led to imagine that the seconds are less than thirty because, if they were more, he would have rounded them off. There is no indication in that statement that he undertook the investigation without Sulaimān.

It is found in some anecdotes that the maximum altitude was observed at Marv and was found to be 75;52°, that the minimum was found to be 28;46°, and hence half the difference between them, which is 23;33°, is the declination. Further, it was related that the altitude of the celestial equator was frequently observed there and was found to be 52;20°, which gives its latitude as 37;40°. If we relate the altitude of the equator to the

maximum altitude, the declination would be 23;32°, but if we relate it to the minimum altitude, the (maximum) declination would be 23;34°. No (MS 88) date or name is mentioned in this story, but since Marv was Manṣūr's home town and the seat of his government, one can well speculate that it was he who conducted those observations.

Further, Muḥammad bin ᶜAli al-Makkī stated in his book, **Al-madkhal ilā ṣināᶜat al-aḥkām** (Introduction to Astrology), that

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the moderns have measured the maximum altitude in the fourth climate and found it to be 78;8°, and that they have found the (maximum) declination to be 23;34°. His book, **Istidārat al-samā' wal-arḍ** (The Spherical Shape of the Heavens and the Earth), testifies that his days were earlier than the time of Sulaimān's observation by more than forty years. The latitude of the fourth climate, according to his statement, must be 35;26°. But it is known that Manṣūr observed only in Nishāpūr and beyond it, north of Marv and Khwārizm, because he only frequented that region, where the latitudes, however, are in excess of the amount mentioned. If his (al-Makkī's) time were nearer, and not prior to that of the Dailamite state, I would have liked to think that he had referred to the observation of Abū al-Faḍl bin al-ᶜAmīd; because he had ordered the construction of a mural quadrant (**libna**) in Rayy (Rhagae), and erected at it a scale, with a base whose diameter was equal to three lightly stretched fingers whose shadow was observed by a thread bisecting it.

Abū al-Faḍl al-Hirawī, in the presence of Abū Jaᶜfar al-Khāzin, observed with it the sun's altitude at noon on Wednesday, the twelfth of Rabīᶜ II (MS 89) year three hundred forty-eight of the Hijra, Zāmdādh (?) day, the 28th of Khurdādh month, year three hundred twenty-eight of Yazdigerd, and found it to be 78;3°. On Thursday, Mārisfand day, he found it to be slightly less than 78;5°; on Friday, Anīrān day, he found it to be 78;6°; on Saturday, Hurmuz day of the month Tīr, he found it to be slightly less than 78;6°; and on Sunday, Bahman

18 day, he found it to be 76;5°. Then he observed its altitude at noon on Friday, the twenty-first of Shawwāl, year three hundred forty-nine of the Hijra, Farvardhīn day of the month of Ādhār, year three hundred twenty-eight of Yazdigerd, and found it to be 30;47°, and on Sunday, Rām day, he found it to be slightly
21 in excess of 30;46°. Therefore the difference between the two solstices is 47;20°, and half of it, which is 23;40°, is the (maximum) declination. Hence the altitude of the first point of Aries is 54;26°, at Rayy,

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and its latitude is 35;34°. The accuracy of the latitude of Rayy was later verified by the observation of Abū Maḥmūd, whose work will be mentioned later on for the preservation of the chronological order, but the amount of the (maximum) decli-
3 nation shocks one's ears, because it is far beyond the generally accepted amount, and totally contradicts what Abū Maḥmūd has found for it.

After that, by order of ᶜAḍud al-Dawla, the declination
6 was observed at Shīrāz, by a ring whose internal diameter is two and a half cubits, i.e. five spans, and whose divisions of parts are at intervals of five minutes. It was conducted by Abū al-Ḥasan ᶜAbd al-Raḥman bin ᶜUmar al-Ṣūfī, in the presence of a group of scholars: Abū Sahl (MS 90) Wījan bin Rustam al-Kūhī,
9 Aḥmad bin Muḥammad bin ᶜAbd al-Jalīl al-Sijẓī, Naẓīf bin Yumn al-Yunānī, Abū al-Qāsim Ghulām Zuḥal, and other scholars.

They observed the altitude of the winter solstice on Wednes-
12 day, the second of Ṣafar, year three hundred fifty-nine of the Hijra, on Bādh day, 22 Ādhār month, year three hundred thirty-eight of the Persians, and found it to be 36;50°; on Thursday,
15 Daibadīn day, they found it to be 36;49°; on Friday, Dīn day, they found it to be 36;50°, and they found it (the minimum altitude) the same on Daibadīn, the 23rd of the month Ādhār, year three hundred and thirty-nine of Yazdigerd.

Then they observed also the altitude of the summer solstice.
18 On Thursday, the eighth of Shaᶜbān, year three hundred fifty-nine of the Hijra, Ard day, 25 of Khurdādh month,

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year three hundred thirty-nine of Yazdigerd, they found it to be slightly less than 83;59°. On Friday, Āshtādh day, they found it to be exactly 83;59°, and on Saturday, Āsimān day, they found it
3 to be slightly less than 83;59°. The difference between 83;59° and 36;49° is 47;10°. Half of it, which is 23;35°, is the maximum declination.

Abū al-Wafā' stated in his Almagest that he made obser-
6 vations for it for several years, and found it to be 23;35°, but he gave no details. We know, however, that most of his observations were made in the days of ᶜIzz al-Dawla, at Bāb al-Tibn (the Straw Gate) in Baghdad, (MS 91) and mainly during the two years three hundred sixty-five and sixty-six of the Hijra, they
9 being three hundred forty-five and six of Yazdigerd. We also know from his Almagest that he found the latitude of Baghdad to be 33;25°, and hence we know that he must have found the maximum altitude to be 80;10°, and its minimum 33;0°.

Abū Ḥāmid al-Ṣaghānī stated, in the book **Qawānīn ᶜilm**
12 **al-hay'a,** that he observed with a ring of diameter six spans, and whose circumference was divided at intervals of five minutes, at Birkat Zalzal, on the western side of Baghdad. He found the greatest declination to be 23;35°, and the latitude of Baghdad to be 33;20°. That was in the year three hundred seventy-four
15 of the Hijra, i.e. three hundred fifty-four of the Persians. We can deduce from this that his findings for the maximum and the minimum altitudes were identical with those found by the sons of Mūsa (bin Shākir).

Furthermore, Sharaf al-Dawla ordered Abū Sahl al-Kūhī to make a new observation. So he constructed

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in Baghdad a house whose lowest part is a segment of a sphere, of diameter twenty-five cubits, and whose center is in the ceiling of the house, at an aperture which admits the rays of the sun to trace the diurnal parallels.
3 Naẓīf bin Yumn informed me, in writing, that the summer solstice was found at the end of the first hour of the night whose

morning was on Saturday, the twenty-eighth of Ṣafar, year three
hundred seventy-eight of the Hijra, Anīrān day, 30 of Khurdādh
6 month, year three hundred (MS 92) fifty-seven of Yazdigerd, and
that the altitude of the Head of Cancer was found to be 80;10°.
Then he added what created doubt and suspicion, namely, his
statement that after a thorough investigation the maximum
declination was found to be identical with what Ptolemy had
9 found, i.e. 23;51,20°, and that the latitude of the city is 33;41,20°.
It is not permissible that the fractions in the (maximum) decli-
nation should hover about one half and one quarter, in all the
observations made, at times near, or remote from, the time of
Abū Sahl. Such differences as appear cannot be attributed to the
12 rotation of the poles of the ecliptic about a point, as was imagined
by Sinān and Abū Jaʿfar al-Khāzin, because that rotation is
irregular. Moreover, later observations have not revealed any
increase in the (maximum) declination. Also, I do not think that
15 the statement was based on actual observation of the winter
solstice. Actually, what was observed was the summer solstice
only, and this was found to be identical with some of the results
mentioned previously, and from it was derived the latitude of the
city which was mentioned above, because the altitude of the
winter solstice should have been 34;27,20°, but instruments very
18 rarely give the seconds accurately. No more details of Abū Sahl's
observation were communicated other than what I have mention-
ed, because Sharaf al-Dawla passed away before that, and the
work was stopped.

Later, by order of Fakhr

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al-Dawla, Abū Maḥmūd (MS 93) Ḥāmid bin al-Khiḍr al-
Khujandī built on Mount Ṭabārak (?) near Rayy, two parallel
walls on the meridian line, at a distance of seven cubits from one
another. He also built on top of them an arch with an aperture,
3 of diameter one span, at its summit. He made the center of the
aperture the center of a sextant of a circle, on the meridian line
between the two walls, of diameter eighty cubits. He laid planks
of wood on it (the sextant), then covered it with a brass surface,
and divided every degree of the complete circuit into three

6 hundred sixty equal parts, thus making the interval between the
parts represent ten seconds. The sun's rays through that aperture
shone on the meridian line.

Abū Maḥmūd constructed a tambourine (?) that would
cover the illuminated spot on the ground and whose center was
9 clearly located by its two intersecting diameters. By applying its
circumference to that of the illuminated spot, he was able to
determine, from the position of the center, the displacement of
the sun from the zenith.

I will now narrate his work, according to his version, in
his treatise, **Fī taṣḥīḥ al-mail** (On Correcting the Declination).
12 When he observed the summer solstice he found the altitude to be
77;57,40° on two consecutive noons. The first was on Saturday,
the fifth of Jumādā I, year (MS 94) three hundred eighty-four of
the Hijra, Hurmuzd day, the first of the month Tīr, year three
15 hundred sixty-three of Yazdigerd. The other was on Sunday,
Bahman day, the 2nd of the month Tīr. From that, he deduced
that the solstice had occurred at the intervening midnight.

18 Then he worked for the winter solstice, but clouds prevented
its observation. However, he obtained the sun's altitude before
the solstice, at midday on Friday, the ninth

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of Dhū al-Qaʿda, three hundred eighty-four of the Hijra,
Āsimān day, 27 of Ādhār month, year three hundred sixty-three
3 of Yazdigerd. It was 30;53,35°. He also obtained it (the altitude)
after the solstice, at midday on Monday, Anīrān day, 30 of
Ādhār month (as) 30;53,32°. From that, he deduced that the
solstice had occurred slightly before midnight on Sunday.

6 Then he wanted to determine the altitudes of the two
solstices, and although he had not obtained until then the mean
and variable motions of the sun and the position of the apogee,
he knew that the available data for these in the zījes of the
moderns do not contradict empirical reality, particularly if he
9 wanted to use them for minute arcs. So he sought the summer
solstice; he found that the interval between it and midday of
that Saturday is twelve hours, and that the sun's motion in that
situation was approximately 0;28,36°, (MS 95) according to

al-Battānī's zīj. Next, he sought the winter solstice; he found that the interval between it and midday of the preceding Friday was thirty-six hours, and that the sun's motion in that situation was 1;31,48°.

Let the circle ABE (Fig.11) represent the ecliptic, and the points G and E be the solstitial points, G for the summer point, and E for the winter point. We join the diameter GE, and we suppose that A is the point whose altitude was observed on Saturday, and B the point for Sunday. Because the altitudes are equal, the arc AG is equal to the arc GB empirically. AG is the arc, 0;28,36°, which he obtained. Also, let D be the point whose altitude was observed on Friday, H (the point observed) on Monday, and K the midpoint of the arc

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HD. Because the altitude of H is less than the altitude of D, the point H is nearer to the solstice, and KH, which is half DH, is equal to 1;31,48°. Now, the difference between the altitudes of D and H, which is three seconds, is equal to the difference between their declinations, and so, if the declination at H differs by three seconds, then the sun must have moved five minutes, according to the motions tabulated in the (above-) mentioned zīj.

With E as pole and ED as radius, we draw the arc DT. Therefore T is the point whose declination, altitude, and displacement from the solstice are respectively equal to those of point D. Hence the difference (MS 96) between the declinations of the points H and T is equal to three seconds, and the arc HT is five minutes. If it (HT) is added to arc KH the sum amounts to 1;36,48°, which is arc KT. He took this to be the displacement of T from the first of Capricorn, I mean E. That is not so, because the required arc TE is part of KT. But KE is half of HT, because the ratio of DE to DT is as the ratio of DK to DH, which is the ratio of a half. Therefore, **alternando et dividendo,** the ratio of KE to KT is the ratio of a half. Hence either he should add TH to twice HK to get the sum TKD, and then take half of it to get the required displacement ET, or he should add half of TH to KH, and thereby he would get an identical result. If he performs one of the two operations, he would obtain 1;34,18° instead of 1;36,48°.

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As he took that amount (1;36,48°) for the arc ET, he then found the difference between it and the arc AG and obtained 1;8,12°, because by drawing AM parallel to GE (we can see that) MT is the difference between ET and AG. Also, since the altitude of T is equal to that of D, and the arc TM is known, then the difference between the declinations of T and M is 0;0,32,30°. So he diminished by it the altitude of T, which is that of D, and there remained 30;53,2,30°, which is the altitude of M, whose displacement from the winter solstice E (MS 97) is equal to the displacement of A from G, the summer solstice. He called the altitude of M the adjusted lesser altitude, and that of A the adjusted bigger altitude. The difference between them is 47;4,37,30°, according to his derivation. But, as I have pointed out, he made a careless mistake in working out this result, and if his computation failed it is because the adjusted lesser altitude is different from what he set out to determine, although the difference is an insensible amount.

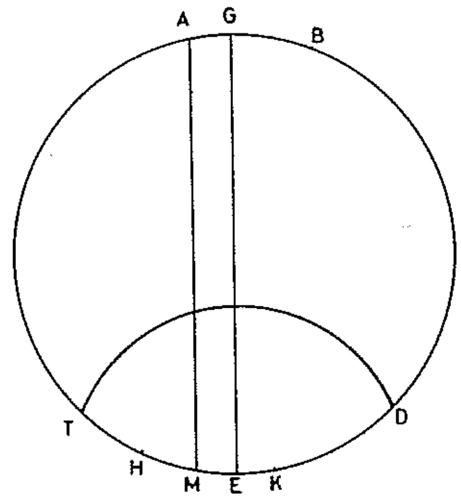

Figure 11

Further, Let HG (Fig. 12) be an arc of the meridian circle which is twice

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the maximum declination in magnitude, and let E be the center of the sphere. We join H to E, and G to H, and we suppose that H represents the summer solstice, and G the winter one. We erect (orthogonal to) the meridian circle a great circle through each one of the two points H and G. ZH is part of the ecliptic and the summer solstice is on it at H. ZG also is part of it, and the winter (MS 98) solstice is at G. Thus he (al-Khujandī) was able to obtain two elevated points at equal distances from H and G respectively. Let these be K and L, where arc HK is equal to arc GL. About the pole of the Whole we construct the parallels KA and LM. A is on the path of K as it crosses the meridian; M is on that of L as it crosses it, and AM is for points between the adjusted altitudes. We draw EB through the middle point of AM, the common line of intersection between the celestial equator and the meridian, and we draw AS and MT parallel to it. We join K to S and L to T. KS and LT are respectively perpendicular to the lines HE and GE, because each is respectively part of the line of intersection between the planes of GZ and ML, and that which is between the planes of HZ and KA. But these planes are perpendicular to the plane of the circle HG, so their common lines of intersection are perpendicular to its plane and to lines which lie in this plane. Now, KS is the sine of HK, SE is the sine of ZK, the complement of KH, LT is the sine of GL, ET is the sine of ZL, the complement of LG, ST is equal to the chord of AM, and all these are known. Also, the triangles EST and EHG are similar, therefore the ratio of ES, the cosine of the distance of one of the two points from the solstice, to ST, the chord between the two adjusted altitudes, is as the ratio of EH, the total sine, to GH, the maximum declination. Hence he multiplied

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chord ST, which is 47;55,26 ° (MS 99), by the total sine, which is 60, and obtained 2875;26, which he retained. He deducted HK,

which is 0;28,36 °, from 90, and there remained 89;31,24 °, which is KZ. SE, its sine, is 59;59,53. He divided the retained product by this sine and obtained 23;57,45,48, which is the sine of the (maximum) declination HB. Its arc is 23;32,21 °.

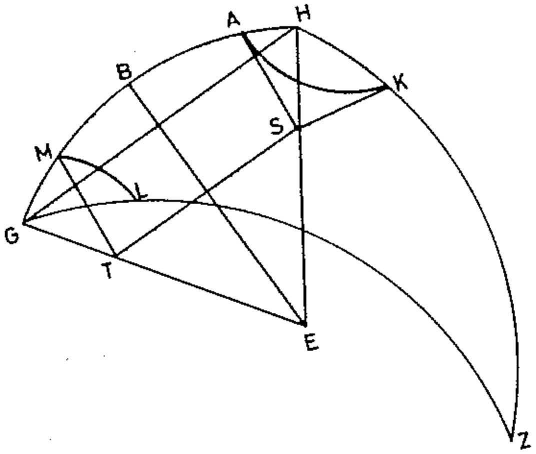

Figure 12

This Fakhrī sextant surpassed all that was made before it and after in grandeur and precision, because Abū Maḥmūd was a unique master of the art of making astrolabes and all other instruments. His result for the amount of the maximum declination (obliquity of the ecliptic) should have been preferred and adopted, and the increase or decrease in the maximum declination should have been based on it, because with his instrument he was able to measure seconds accurately, and so, what accuracy of minutes! But Abū Maḥmūd (MS 100) informed me orally of a discrepancy in the observation. The aperture

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above the arch moved downwards about a span, and he was not

able to set it back in its proper position. As evidence of that, we note that the amount of the (maximum) declination which he found is less than the amount found about his time, the amounts which have already been mentioned, and those which will follow.

If ABGD (Fig. 13) is the meridian circle with center E, and A on it the zenith, B the meridian passage of the summer solstice, and G the meridian passage of the winter solstice, then arc BG is twice the declination. Let DH represent the arc of which the Fakhrī Sextant was made, then E is the position of the aperture above the arch, because it is the center of the sextant, which is situated empirically at the center of the Whole. The summer ray penetrates through it along BEZ, and the winter ray along GEH. Hence arc HZ is twice the maximum declination, because of the similitude between the arcs BG and HZ.

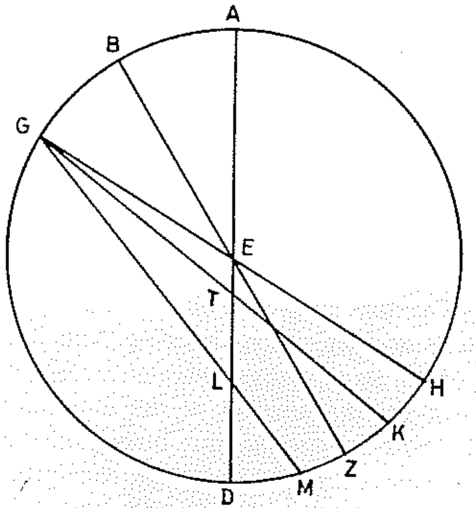

Figure 13

If we suppose, as Abū Maḥmūd has mentioned, that the aperture dropped to T, then the ray of the winter solstice would enter it along GTK, and KZ would be equal to twice the

observed declination which is less than HZ, the true amount. The more the aperture moves downwards, the more is the diminution in the amount of (MS 101) the declination, and if its depression becomes excessive, it is quite possible that the winter ray might coincide with the true summer ray, and then the observed declination will cease to exist. Or it may even go below that position, like the ray GLM, which penetrates through the aperture at L. But this will make DM, the complement of the winter altitude, less than DZ, the complement of the summer altitude, and that is impossible.

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This is why an observer should keep alert, constantly scrutinizing his work, promoting his self-criticism, moderating his self-admiration, and pursuing his researches without impatience or boredom.

As far as I know, this is the last observation which was made for the determination of the maximum declination.

But concerning myself, I am deeply interested in such observations, for they are more preferable to me than all other ambitions. I feel that I have been prevented from pursuing them and from making full (MS 102) use of my competence in that field. I intended to conduct a series of observations during the two years three hundred eighty-four and five of the Hijra, and had fully prepared for them a circle of diameter fifteen cubits, and all other supplementary equipment. But I was not able to carry on the observation, except for the maximum altitude at a village, west of the Jayḥūn and south of the city of Khwārizm, and for the altitude of azimuth zero. I related this previously, when I derived the latitude of that place. But the declination is the difference between the maximum altitude and the colatitude of the place of observation, and hence what was then obtained for the (maximum) declination is 23;35,45°.

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Also, in that previous figure (Fig. 6), where these two altitudes were used, if we drop from E a perpendicular to TZ, it will be equal to the sine of the declination of the parallel, which

is, at that time, the solstitial parallel. The ratio of TW to WY is as the ratio of YE to ZE, hence ZE is known. Also the ratio of ZE to the perpendicular (drawn) from E to ZT is as the ratio of TY to TW. Hence that perpendicular is known, and it is the sine of the maximum declination.

After that day, disturbances broke out between the two lords of Khwārizm, and the work had to be stopped because I had to take shelter, and I then sought safety by emigrating from the fatherland. (MS 103) After I had barely settled down for a few years, I was permitted by the Lord of Time to go back home, but I was compelled to participate in worldly affairs, which excited the envy of fools, but made the wise pity me.

Later on, I was able to devote some time to making observations, in the days of the martyr prince, Abū al-ᶜAbbās, the Khwārizmshāh, may God give him the power for a luminous plea (on the Day of Judgment)! I obtained the maximum altitude and the altitude of azimuth zero, which were mentioned previously, when the latitude of that place was derived. However, by the end of that year, the country was a scene of disturbances and extermination, but I took hardly any notice of them, because I had been working with my mind for a long time. The peace and security which followed that disturbed period were precarious, and did not permit a return to the former state of things, and to activities that are preferable to people like me.

Further, the maximum altitude at Jurjāniya was 71;18°, and the complement of

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the latitude, which we presented in that figure (Fig. 6), was 47;42,10°. Hence the difference between them, which is 23;35,50°, is the maximum declination. On the other hand, if we multiply WY by YE, we get the product 8,897,635,464 fourths and if we divide (MS 104) (this product) by TW, we get for ZE 116897 seconds. But because we need to multiply ZE by what we divided by, namely TW, we (simply) divide by TY, leaving out the division by TW for brevity. We divide by TY what resulted from multiplying WY by YE. There comes out 24;1,5, the perpendicular dropped from E to ZT. Its arc (sine), 23;35,50°, is the maximum declination.

After that, it happened that I observed the maximum altitude of the summer solstice at Ghazna, and found it to be eighty degrees in the two years three hundred eighty-eight and nine of Yazdigerd. Also, in the year three hundred eighty-eight of Yazdigerd I found the midday altitude of the winter solstice to be thirty-two degrees and [five] sixths. Hence the maximum declination is 23;35°, and the latitude of Ghazna is 33;35°. God helps to the right insight!

What has been transmitted from India in the Indian zīj, known as the Sindhind, gives the amount of the (maximum) declination as exactly twenty-four degrees. However, a researcher, reading about their works, finds that they are far from being critical investigations, and does not have confidence in any claim to precision in their observations. But, as the people

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are so far away and very reticent, and as they hold fast to their superficial knowledge of little things, and as our laymen believe in their wisdom, (even) though they (the Indians) do not have it, and because those works are simple in comparison with well-investigated works, they have had many fanatical admirers who overlook a glaring fact, and do not mind plagiarizing to attribute to them other people's works.

One of them (the admirers) is Muḥammad bin ᶜAli al-Makkī who stated in his book, **al-Madkhal ilā aḥkām** (MS 105) **al-nujūm fil-layl,** that this difference arises because they have referred their observations to the center of the universe, while other people's observations have been referred to the surface of the earth. Those who have heard that statement have taken its validity for granted, without melting the ore and extracting the gold from it. I must assay all aspects of this statement, because I do not refuse to accept the truth from any source, wherever I can find it.

Let A (Fig. 14) be the zenith of the observer, H his position on the surface of the earth, E the center of the universe, and ABG a part of the meridian circle. Let B on this circle be the meridian passage of the summer solstice, and G the passage of the winter

solstice. Then the arc BG between the two solstices is twice the maximum declination. The researchers whose works we have enumerated have found this arc by the two bounding lines HB and HG, but what was reported from India is the existence of the arc bounded by the lines EB and EG. There is no way of finding it by actual measurement, because one cannot reach the center of the universe, but one can transform to it from H if both HE and EA are known.

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We produce BH and drop on it the perpendicular EZ, which is the sine of angle ZBE in the circle whose radius is EB. We also produce GH and drop on it from E a perpendicular which cannot coincide with EZ. Because, if it is possible, let it be EZT. Then in the triangle (MS 106) HZT, two angles, at T and at Z, are right angles, and this is impossible. If the angle at Z is a right angle, then the angle at T is necessarily acute. Thus the perpendicular dropped from E on GH must fall between the points T and H. Let it be EKM; then it (EM) is the sine of angle HGE in a circle identical with that circle. EK is the hypotenuse of a right triangle whose legs are the equals of EZ and ZK. So it (EK) is greater than EZ. But EK is a part of EM. So EM is much greater than EZ, and consequently angle G is greater than angle B. Also, angle AHB, the actual coaltitude of the summer solstice, minus angle AEB, its altitude relative to the center, is equal to angle HBE, which is the parallax angle. Similarly, angle AHG, the coaltitude of the winter solstice, minus angle AEG, is equal to angle AGE. Thus the effect of parallax is to diminish the coaltitude, and so if we subtract them (the angles of parallax) from them (the actual coaltitudes), the result will be less than their difference without the diminution due to parallax, by the difference between the angles B and G, because angle G is greater than angle B.

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Therefore the (maximum) declination as measured by the Indians should have been less than the declination measured by other observers.

If they reverse their argument and pretend that observations in India have been adjusted for parallax, and that the observations of others have neither been adjusted for it nor referred to the surface of the earth, (MS 107) because the measurements of the latter were made by rings whose centers are tantamount to the center of the universe, while Indian observations are made with shadows, then this would be their last assumption, and we shall concede to them its validity until we assay it with their own assay. There is no divergence of views between them (the admirers of the Indians) and others about the maximum amount of parallax and that it does not amount to one half of one tenth of a degree.

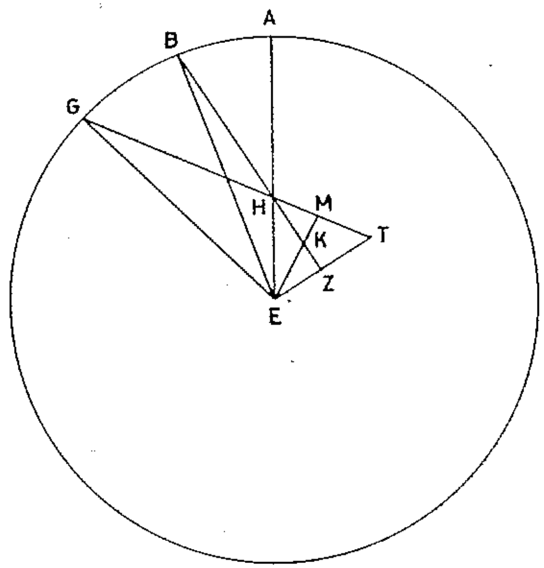

Figure 14

But the difference between their (Indian and other) estimates of the declination (obliquity of the ecliptic) is one quarter plus one sixth of a degree, and the latitude of the moon according to them (the Indians) is less than Ptolemy's estimate by half a degree. So whatever they assume for India regarding the center E, their

claim for the declination is false, and if they (the admirers) assume for them the point H, then the amount of the latitude of the moon contradicts their assumption. So they have to retract and concede that they made their observations like others.

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Further, let us make the observation under the parallel of declination of the summer solstice, so that B (Fig. 15) shall coincide with the zenith (A), then twice the maximum declination is the coaltitude of the winter solstice, I mean AG, (MS 108) which is empirically given by angle AHG. But, if referred to the center E, it is given by angle AEG, which is less than AHG by angle HGE. So it (the maximum declination) is less again, not more. But the diminution is so small that they had better not attach themselves to it (the idea) if they are sensible.

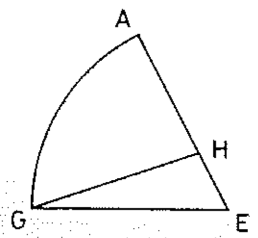

Figure 15

Also, let us make the observation on the equator, then A is midway between B and G (Fig. 16), AB is the coaltitude of the summer solstice, AG is the coaltitude of the winter solstice, and their sum is twice the maximum declination. If it is transformed from the place of observation to the center, it will be given by the magnitude of angle BEG which is less than angle BHG by the sum of the two angles at B and at G. The conclusion is the same, no matter where the observation is made, and so there is no substance in that assumption.

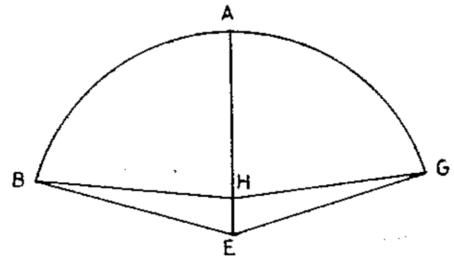

Figure 16

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Now all the testimonies that we have adduced point out collectively that the maximum declination is twenty-three degrees, plus one third and one quarter of a degree. The slight excess or defect in some of the estimations is due to the instrument (of observation). In particular, the defect in the amount found by Abū Maḥmūd, and the excess in the amount found by Abū Sahl, which was reported by Naẓif, are both due to the instruments, because we have found that during al-Khujandī's year of observation the fractional parts in the declination are neither less than one third plus a quarter nor more than that.

We present in tables what was mentioned above, so that its repetition may serve as a check against errors of transcription, and to reduce the length of a sentence so that the eye can see it all at the same time.

The difference between two maximum altitudes at two cities should necessarily be equal to the difference between their latitudes. However, this equality does not hold in what is presented in the table, because the maximum altitude depends on the declination, and the latitude of a city depends on the difference between the maximum and the minimum altitudes. Therefore the different estimates of the declination may possibly have introduced an error in one or both altitudes. God leads to the right insight! (MS 110)

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IV. ON THE RELATIONSHIP BETWEEN A CITY'S LATITUDE AND THE TOTAL OR PARTIAL DECLINATION

We have already discussed, separately, the determination of the latitude of a city, and the total declination, each of them independently of the other. But both are like related syntactical cases, where one of them can be studied with the aid of the other, and a study of their relationship may help us to obtain some valuable knowledge of this art. We shall now devote our attention to this subject.

I say: If we are given either the maximum declination, or the partial declination, I mean a parallel of declination other than the solstitial parallel, and if we would like to know only the latitude of a city, then we observe an altitude of the sun of known azimuth, and this will enable us to determine the latitude of the city. It (the altitude) may be on the meridian, or it may be on the east-west line, or it may be displaced from it to the south or the north.

In the case of a meridian altitude, it may be south of the zenith, or it may be north of the zenith, or it may be right overhead.

Let the circle ABGD, with center E, represent the meridian circle (Fig. 17), A the zenith, B the south point, and EZ the common line of intersection between its plane and the plane of the celestial equator. Then AZ is the (MS 111) required latitude. If the meridian altitude is south of the zenith, I mean if it is measured from point B, and if the sun's declination is southern

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like ZH, whether partial or total, then the altitude is given by BH. So, if we take the difference between AH, the coaltitude, and ZH which is the sun's declination, we get AZ which is the latitude of the city.

Again, if the altitude is measured from B, the south point, but the sun's declination is northern, like ZT, then the meridian altitude is TB. So, if we add the coaltitude AT to the declination ZT, then the sum, which is AZ, is the latitude.

Further, if the sun's declination is zero, then the altitude is given by ZB, and the coaltitude, which is AZ, is the latitude of the city.

Also, if the altitude is a complete quadrant, and if the sun has a declination, then its declination, which is given by AZ, is equal to the latitude of the city.

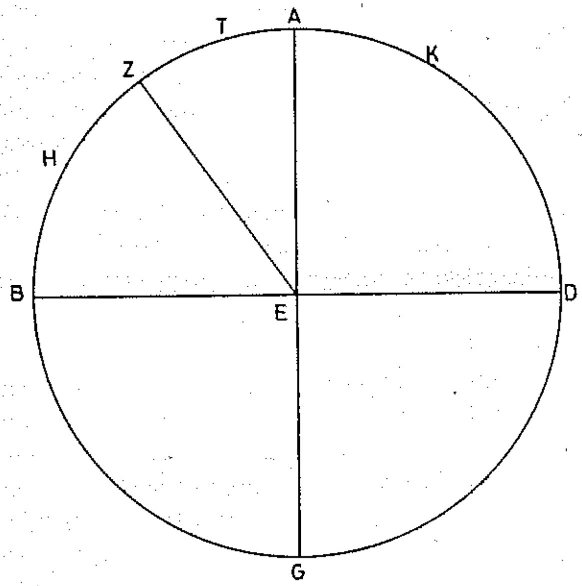

Figure 17

But if the meridian altitude is measured from the north point D and is given by DK, then we subtract the coaltitude AK from the declination ZK, and there remains AZ, the latitude of the city.

Further, if the sun's declination is zero and its altitude is a

complete quadrant, or if it has a (non-zero) declination and its altitude is equal to the complement of its declination, then we are on the terrestrial equator.

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For example, on the day of writing this chapter, Tuesday, the first of Jumādā II, the year four hundred nine of the Hijra,
3 I was in Jayfūr, a village adjacent to Kābul. I had a keen interest in making observations for determining the latitudes of those places, though I was in strain and agony. I think that even Noah and Lot - peace be granted to them! - did not suffer such agony, and I do hope, with God's munificence, to be their third in
6 receiving His mercy for my salvation. I had no instrument then for measuring the altitude, and was not in possession of any of the material from which one might be made. So I drew an arc of a circle on the back of the computation board, and divided each degree into six equal parts, so that each part represented an
9 interval of ten minutes, and suspended it vertically, by plumb lines. The altitude south of the zenith was found to be 45;0°, but according to al-Battānī's zīj, when the sun's position is 26;36° of Libra, its declination is 10;19° southwards. So I added this to
12 the observed altitude, and obtained the sum 55;19°, which is the colatitude of Kābul, and hence the latitude itself is 34;41°.

Another example: Abū al-Faḍl bin al-ᶜAmīd had given orders for measuring the meridian altitude on Saturday, the
15 twelfth of Shaᶜbān, year three hundred forty-nine of the Hijra in the city of Kāshān, when the sun's position was 18;37° of Libra. According to the Ṣafāyiḥ zīj, which was prepared for him by Abū Jaᶜfar (MS 113) al-Khāzin, it (the altitude) was found to be 50;0°, and the

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sun's declination was 7;20°. So the altitude of the equinox was 57;20°, and the latitude of Kāshān is 32;40°. It is known that there was confusion in (the determination of) the altitude, be-
3 cause Kāshān lies between Iṣfahān, whose latitude is greater than that amount, and Rayy the latitude of which is also greater.

In case the order is reversed, that is, if we obtain the

meridian altitude and we wish to determine the declination, having already determined the latitude, then we have to consider
6 the following cases: (a) If the altitude measured from the south point is equal to the colatitude, like ZB, then the sun is on the celestial equator and its declination is zero. (b) If it is less than the colatitude, like BH, then the difference between them, I
9 mean ZH, is the declination, which is due south. (c) If it is greater than the colatitude, like BT or AB, then the difference between them, I mean TZ or AZ, respectively, is the declination, which is due north. (d) If the altitude is measured from the north point,
12 like DK, then the sum of the latitude AZ and the coaltitude AK, which is equal to ZK, is the declination.

For example, I found (observed) at Jurjāniya, at the Emirate
15 House, the noon altitude on Monday, the eleventh of Rabīᶜ II, in the year four hundred seven of the Hijra, Ābān day, 10 Mihr month, year three hundred eighty-five of Yazdigerd (MS 114),
18 the seventeenth day of Aylūl, year (one) thousand three hundred twenty-seven of Alexander, and I found it to be 47;42° (or 47;44°). Since this is greater than the colatitude of Jurjāniya, which is 47;43°, the difference between them, which is 0;1°, is the sun's declination to the north. The autumnal equinox occurred

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after midday by one equinoctial hour. I made this observation the basis for the discussion of the mean solar motion in the **Kitāb**
3 **al-taṭrīq ilā taḥqīq ḥarakāt al-shams** (A Book on the Method of Investigating the Motion of the Sun).

If both the latitude of a city and the declination of the sun are known, and we want to determine the meridian altitude,
6 then we subtract the southern declination from the colatitude, or add the northern declination to it, and we obtain the meridian altitude from the south point. But if the resultant (difference or sum) exceeds a quadrant, like the arc BAK, then we subtract that resultant from a hundred and eighty degrees, which is arc
9 BAD, and there remains DK, which is the meridian altitude from the north point.

If the observed altitude is on the circle of azimuth zero and it is required to find the latitude, then we reproduce from

the corresponding figure, mentioned previously, the part and the symbols which we need.

Let LO (Fig. 18) represent the sine of the observed altitude on the east-west line EB. If we drop (MS 115) OK perpendicular to LF, then OK is the sine of the sun's declination. Because angle LFO is equal to the colatitude and angle FOL is a right angle, therefore the remaining angle FLO is equal to the latitude. Now, the ratio of LO, the sine of the observed altitude, to OK, the sine of the solar declination, is as the ratio of the sine of angle LKO, the right angle, to the sine of angle OLK, the latitude

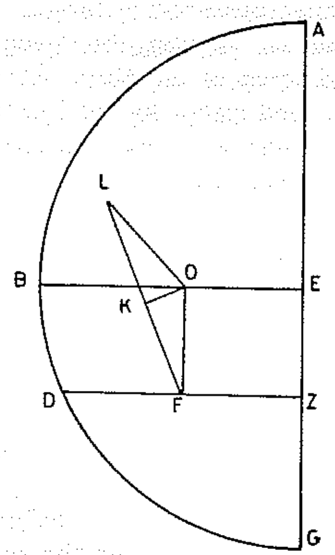

Figure 18

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of the city. Therefore, if we multiply the sine of the sun's declination by the total sine, and then divide the product by the sine of the altitude of azimuth zero, we obtain the quotient which is the sine of the altitude.

If, with the altitude, the known datum is the latitude of the city, and it is required to find the sun's declination, then we have that the ratio of LO, the sine of the altitude, to the required OK is as the ratio of the sine of angle LKO, the right angle, to the sine of angle OLK, the latitude of the city. Therefore, if we multiply the sine of the altitude of azimuth zero by the sine of the latitude, and then divide this product by the total sine, we obtain the quotient which is the sine of the sun's northern declination, because this altitude (of azimuth zero) can only exist for northern parallels of declination.

If we suppose that the sun's declination is known, and it is required to find the altitude of azimuth zero at a city of known latitude, then we multiply the sine of the sun's declination by the total sine and divide this product by the sine of the latitude of the city. The quotient obtained thereby is the sine of the altitude of azimuth zero. (MS 116)

If the azimuth of the observed altitude deviates from the east-west line on one of its two sides, and if, by hypothesis, the sun's declination is known, and it is required to find the latitude of the city,

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let EM (Fig. 19) represent the azimuth line whose deviation from the east-west line is given by the known arc BM, (determined) by observation. Then the ratio of EO, the cosine of the observed altitude, to OC the share of the azimuth, is as the ratio of EM, the total sine, to the sine of BM.

Therefore, if we multiply the cosine of the altitude by the sine of the azimuth, retain the product, and then divide it by the total sine, the quotient we get is a measure of OC, the share of the azimuth. We join C to L, and if we drop CK perpendicular to LF, then CK is (MS 117) equal to the sine of the declination. Also, CL is known because it is the hypotenuse in the right triangle whose sides LO and OC are known. But its (LO's) ratio to OC is as the ratio of the sine of angle LOC, the right angle, to the sine of angle OLC.

So if we square both the sine of the observed altitude and the share of the azimuth, and if we divide the retained product

by the (square) root of the sum of the two squares, the quotient is the sine of angle OLC. The arc of this sine is the first arc.

Further, the ratio of CK to CL is as the ratio of the sine of angle CLK to the sine of angle CKL, the right angle. Therefore, if we multiply the sine of the declination by the total sine, and divide the product by that (square) root, the quotient will be the sine of angle CLK. The arc of this sine is the second arc.

In the case of a southern azimuth and a southern declination, the difference between the first arc and the second arc is the latitude of the city.

In the case of a southern azimuth and a northern declination, the sum of the two arcs is the latitude of the city.

But in the case of a northern azimuth, the supplement of the sum of the two arcs gives the

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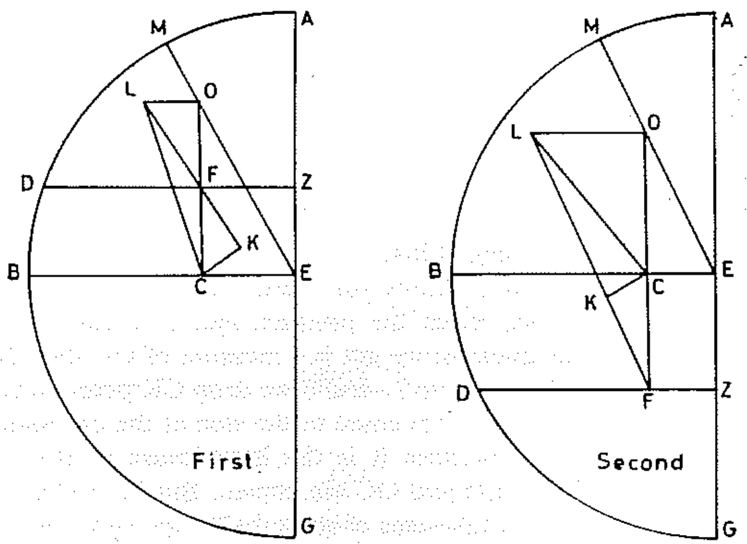

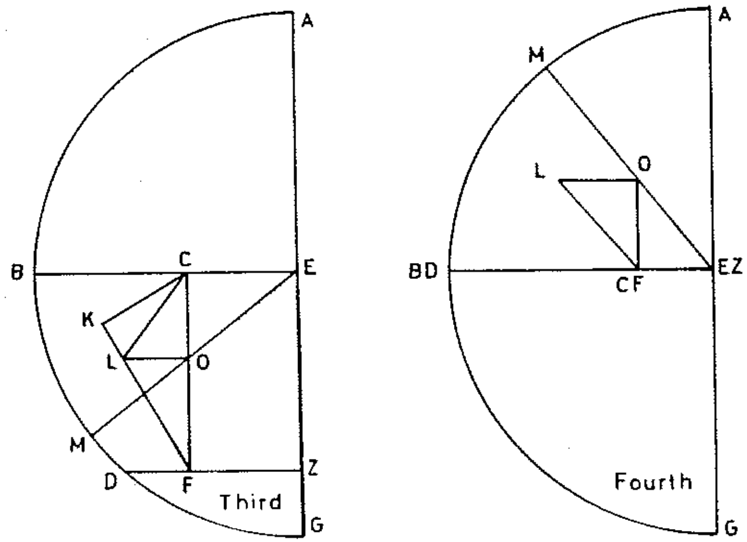

Figure 19

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latitude of the city, because the sum, in the third part of the figure, is the obtuse angle OLK, which is the supplement of angle OLF, the latitude of the city.

If the sun's declination is zero, as in the fourth part of the figure, (MS 118) then what comes out for the first arc is a measure of the latitude of the city.(MS 119)

If by supposition in this case of an altitude with a known azimuth, the latitude is known and it is required to find the declination, we evaluate the share of the azimuth to make OC known. (MS 120) The ratio of LO to OF is as the ratio of the sine of angle OFL, the colatitude, to the sine of angle OLF, the latitude. So if we multiply the sine of the altitude by the sine of the latitude of the city, and then divide this product by the cosine of the latitude, and take the difference between the share of a southern azimuth and the quotient, or find the sum of the share of a northern azimuth and the quotient, then the result of either

operation will be CF. But its ratio to CK is as the ratio of the sine of angle CKF, the right angle, to the sine of angle KFC, which is a measure of the colatitude. Therefore, if we multiply that result, CF, by the cosine of the latitude of the city and then divide the product by the total sine, the quotient will be the sine of the sun's declination.

If we are given the city's latitude and the solar declination as two known data and we are required to find the unknown azimuth when the altitude is known, or we are required to find the unknown altitude when the azimuth is known, then, for the first case, we say: CK, the sine of the declination, is known, and angle KFC is the colatitude. And the ratio of CF to CK is as the ratio of the sine of angle CKF, the right angle, to the sine of angle KFC. Therefore, if we multiply the sine of the solar declination by the total sine, and then divide the product by the cosine of the latitude of (MS 121) the city, the quotient we get is a measure of CF, which is to be retained. It is the hypotenuse of the right triangle whose sides are CK and KF. Therefore, if we square both that quotient (CF) and the sine of the sun's declination (CK),

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then the (square) root of the difference between the two squares is a measure of KF. But the ratio of KF to KC is as the ratio of FO to OL. So if we multiply this root (KF) by the sine of the given altitude (OL) and then divide the product by the sine of the solar declination (KC), the quotient we get is a measure of OF. The difference between it and the retained amount (CF) gives the share of the azimuth for a northern declination, but their sum for a southern declination is the share of the azimuth, the ratio of which to the cosine of the altitude is as the ratio of the sine of the azimuth to the total sine. So if we multiply the share of the azimuth by the total sine and divide the product by the cosine of the altitude, then the quotient we get is the sine of the azimuth.

For the second case, let AZGD (Fig. 20) represent the meridian circle, DBZ half the celestial equator with pole E, ABG the horizon with pole S. Also, let the sun be at L, and

let us draw through it SLM and ELT, where LT is its declination, LM is its altitude, and BM is its azimuth. By supposition the azimuth is known but the altitude is unknown, hence BM and MA are known. Also TL and SD are known. About pole H, and with distance equal to the side of the square, we draw the (great) circle KCO. It follows that BO is equal (MS 122) to MA, and that (arc) CK is a measure of (spherical) angle H. But the ratio of the sine of BO to the sine of OC, the complement of CK, is as the ratio of the sine of BG, the quadrant, to the sine of GZ, the complement of EG. Therefore, if we multiply the cosine of the distance of the azimuth from the east-west line by the cosine of the latitude, and then divide the product by the total sine, the quotient we get is the cosine of angle H. We find its arc cosine, and there results angle H. Also, the ratio of the sine of HL to the sine of LT is as the ratio of the sine of angle LTH, the right angle, to the sine of angle H. Therefore, if we multiply the sine of the sun's declination by the total sine, and divide the product by the sine of angle H, the quotient we get is a sine,

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and its arc sine, LH, is the first arc. Further, the ratio of the sine of HS to the sine of SD is as the ratio of the sine of angle SDH, the right angle, to the sine of angle H. Therefore, if we multiply the sine of the latitude of the city by the total sine, and divide the product by the sine of angle H, the quotient we get is a sine, and its arc sine SH, is the second arc. If the declination is southern, we add the two arcs, the first and the second, but if it is a northern declination, we take the difference between them. The result of each operation gives, respectively, the coaltitude of that given azimuth. If the sun (MS 123) is on the celestial equator, then the first arc is a measure of that coaltitude.

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We have presented above a demonstration for the determination of the latitude of a city, if two altitudes and their azimuths are known for two points (MS 124) on the same parallel of declination. From that same demonstration, the declination of the parallel can be determined.

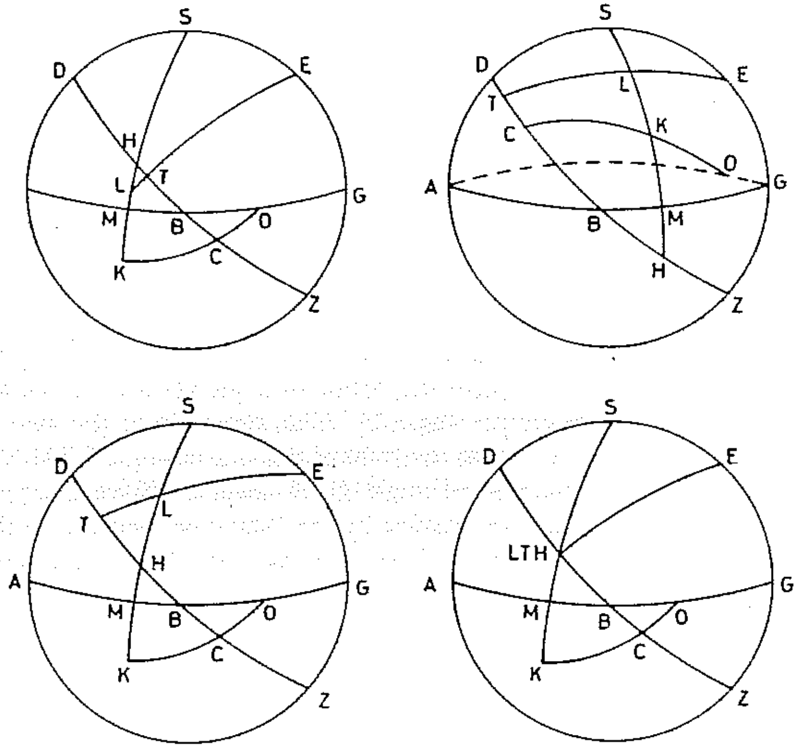

Figure 20

Let us produce by a study of those diagrams, the appropriate diagram for my observations which I gave as an example. I say: If the sides TW and WY of the triangle TWY (Fig. 21) have become known, and because KΘ, the share of the bigger azimuth is known, and the ratio of TW to WY is as the ratio of TK to KN, hence KN is known. But KΘ is known and ΘN is the difference between KΘ and KN for a southern azimuth, and (it is) their sum for a northern azimuth. Therefore ΘN, which is equal to the sine of the rising amplitude, is known. We drop ΘS perpendicular to TN, then ΘS will be equal to the sine of the

declination, because it is equal to the perpendicular issuing from E to the plane of the parallel of declination, and it falls between the two centers (E and the center of the parallel). Also, the triangles ΘSN, NKT, and TWY are similar, and hence the ratio of NΘ to ΘS is as the ratio of TY to TW. Therefore, if we multiply the sine of the rising amplitude by the difference between the sines of the two altitudes, and then divide the product by the hypotenuse, the quotient we get is the sine of the solar declination.

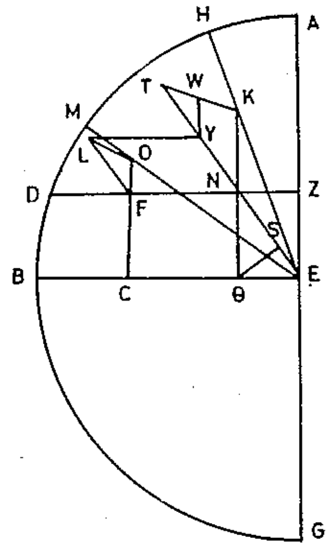

Figure 21

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In the case of the two altitudes and their azimuths, which I had observed and whose magnitudes were given in an example previously, we obtained there the magnitudes of TW, WY, and KΘ. If we multiply the difference between the shares of the two

azimuths, which is 5;40,42, by the sine of the bigger altitude, which is 21;39,54, the product is 1,594,353,348 fourths. If we divide this product by the difference between the two sines, (MS 126) which is 6;18,16, the quotient we get is 19;30,48. The difference between this quotient and the share of the bigger azimuth is 32;10,47. If we multiply this difference by the difference between the sines of the two altitudes, the product we get is 2,629,263,512 fourths. If we divide it (the fourths) by the seconds of the hypotenuse, which is 30,545, the quotient we get is 23;54,38, which is the sine of the sun's (maximum) declination. The arc of this sine is 23;29,6°.

For our present purpose, this is not as reliable as the latitude of the city, because the former involves a series of computations while the latter depends on observation without any computation. However, I have checked it by several methods: one of them is my inductive reading of the maximum degrees for equal declination on opposite sides. I added the two observed meridian altitudes, divided the sum by two, and the halves came out 47;43°, which is equal to the colatitude of the city.

For example, on Sunday, the twenty-sixth of Rabīʿ I, year four hundred seven of the Hijra, Ard day, 25 Tīr month, year three hundred eighty-five of Yazdigerd, the meridian altitude was 53;35°, and on the preceding Saturday the meridian altitude was 53;58°. According to

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Ḥabash's zīj, the sun was then in the sign of Virgo at (MS 127) 15;22°. If we diminish this by eleven minutes, which is the difference between what I have found by observation and the computation by this zīj, then the sun must have been at 15;11° in the sign of Virgo. According to that, the altitude of half (i.e. the middle of) this sign at the meridian of Jurjāniya is 53;36°. Also, on Tuesday, the twenty-sixth of Rabīʿ II, on Ard day, 25 of Mihr month, the actual meridian altitude was 41;53°, and on the succeeding Wednesday the meridian altitude was 41;30°. According to Ḥabash's zīj, the sun was then in the sign of Libra at 15;2°, and if we diminish this by the eleven minutes, the sun must have been in it at 14;51°. Therefore the altitude of

the middle of this sign, at the meridian of Jurjāniya, is 41;52°. If we add the two altitudes, the sum is 94;28°. Half this sum, which is 47;14°, is the latitude of the city. (The differences in) all these considerations are of the order of one minute.

Also, we can make a square board, and put a mark at its center to set up on it a gnomon which we can graduate **ad libitum** in any one of the following scales of shadows (or cotangents): In twelve parts to be in the scale of digits, in six and a half parts to be in the scale of feet, or in sixty parts to be in the sexagesimal scale. Then we open a compass for a radius equal to the tangent of the sun's declination (MS 128) at the time of observation, and at that mark as center and with that radius we draw a circle on the board. Then we set up the gnomon at the center, normal to the board, and set up the board across the meridian line, I mean that one side shall coincide with the east-west line, putting the gnomon on the side of

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the pole of the sun's declination. We tilt the board very slowly, rotating it about that side without making it lose coincidence with, or parallelism to, the east-west line, until the tip of the shadow falls on the circumference of that inscribed circle. The angle then formed between the board and the plane of the horizon is equal to the colatitude of the city. That is so, because the parallels, with respect to the celestial equator, are analogous to the almucantars with respect to the horizon. If we reckon a parallel of declination as an altitude (from the equator), then the parallel would be its almucantar. But the shadow of one and the same almucantar is the same; therefore, when the tip of the shadow falls on the circumference described for the declination's almucantar, the plane of the board falls empirically in the plane of the equator, and the gnomon lies in the direction of the polar axis. Hence the angle of that stated amount is formed, because it is the angle formed by the intersection of the horizon with the equator. This is the case if the meridian line is known. (MS 129) But in case it is unknown, we seek a position for setting up the board such that the tip of the shadow falls on the circumference of the circle. When that position is found, we drop a plumb

line from the midpoint of its upper edge and join its projection to the midpoint of the lower side. The line thus drawn is the meridian line.

These are the methods, dependent upon simultaneous observations of altitudes and their azimuths, for the determination of a city's latitude from the sun's declination. Since the parallels of declination vary with latitude, it is possible to obtain from concomitant phenomena some data which help to derive one of those required two (declination or latitude) from the other. The data are the rising amplitude and half the arc of daylight. If one of these two is observed and one of the required two is also known, then the other can be derived.

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Let ABGD represent the meridian circle, AEG the celestial equator with pole T, and BED the horizon of a given inhabited place (Fig. 22). Also, let H be the rising point, then HE is its rising amplitude which is investigated, by supposition, at a city of known latitude DT. The ratio of the sine of EH, the rising amplitude, to the sine of HK, the declination of the part, is as the ratio of the sine of ED, the quadrant, to the sine of DG, the complement of the latitude of the city. Therefore, if we multiply the sine of the rising amplitude by the cosine of the latitude, and then divide the product by the total sine, then the quotient obtained thereby is the sine of the partial declination. Also, from the preceding proportion, if we multiply the sine of the partial declination by the total sine and divide the product by the sine of the rising amplitude, then the quotient obtained thereby is the cosine of the city's latitude.

If its daylight arc is measured by water or sand instruments (by a clepsydra or a sand-glass), then KA is its (the arc's) half, and KE is the equation of daylight. If, in addition, the latitude of the city is known but the declination is unknown, then we extend the arcs HDL, HTMO, and EAO, and with H as pole and distance equal to the side of the square, we draw the arc LMSC. It follows that AC is equal to EK, SA is equal to DT, and TM is equal to HK. But the ratio of the sine of TS, which is equal to DG, to the sine of SM, is as the ratio of the sine of

TA, the quadrant, to the sine of AO, which is equal to GK. So SM is known. Hence its complement, SC, is (also) known, and the ratio of the sine of SC to the sine of AS, which is equal to EK, is as the ratio of the sine of ST, which is equal to DG, to the sine of MT, which is equal to HK. So if we multiply the cosine of the latitude of the city by the cosine (MS 131) of the equation of daylight, and divide the product by the total sine

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we obtain thereby a sine. We find its arc cosine, take the sine of the latter, and divide the result into the product of the sine of the equation of daylight and the cosine of the latitude of the city. The quotient we obtain is the sine of the partial declination.

If it is supposed that the partial declination is known, but the city's latitude is unknown, we say that the ratio of the sine of TH to the sine of DH is as the ratio of the sine of TK, the quadrant, to the sine of KG. So DH is known. Also, the ratio of the sine of HE, its complement, to the sine of EK is as the ratio of the sine of TH to the sine of TD. So TD is known. Therefore, if we multiply the cosine of the partial declination by the cosine of the equation of daylight, and we divide the product by the total sine, the quotient is a sine. We find its arc sine and subtract this arc from ninety; then we divide the sine of the remainder into the product of the cosine of the partial declination and the sine of the equation of daylight. The quotient obtained thereby is the sine of the city's latitude.

If both the rising amplitude and the equation of daylight have been observed, but both the latitude of the city and the partial declination are unknown, (we say that) the ratio of the sine of TH to the sine of HD is as the ratio of the sine of TK, the quadrant, to the sine of KG. Therefore, if we multiply the cosine of the rising amplitude by the total sine, and divide the product by the cosine of the equation of daylight, the quotient obtained thereby is the cosine of the partial declination. And because the ratio of the sine of TH to the sine of TD is as the ratio of (MS 132) the sine of HE to the sine of EK, if we multiply the cosine of the declination by the sine of the equation of

daylight and divide the product by the sine of the rising amplitude, the quotient we obtain thereby is the sine of the latitude of the city.

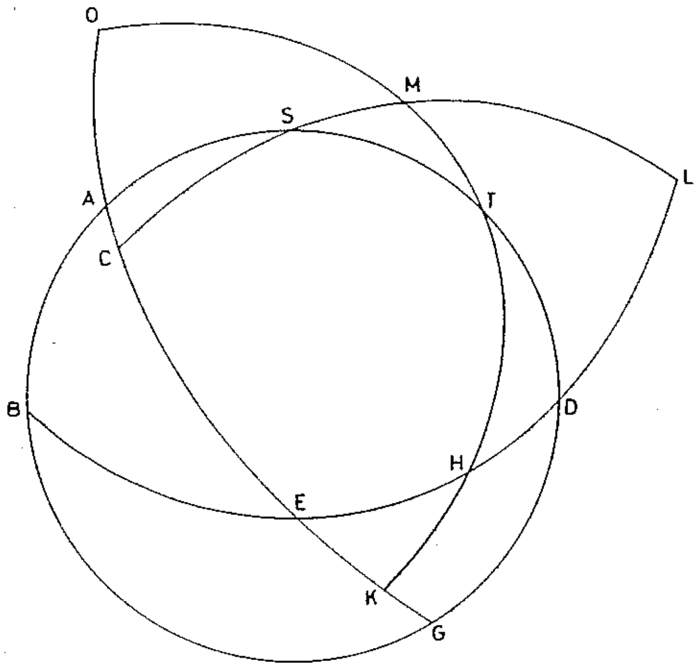

Figure 22

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That was a full demonstration of the different aspects of
that problem. The subject of the climates is closely related to
those aspects, and it is necessary for our plan, because you can
hardly find two manuscripts with identical versions about the
3 latitudinal ranges of the climates. The narratives about them
are named after their representatives, just as different philosophies or theologies are named after their representatives. They
are not things which exist as a result of experimental observation,
and thus there is no reason for different classifications. They are

not propounded by an abstract theory, or by contemplation, to
6 warrant the application of different methods of approach. They
are simply built up by agreement upon a common basis. I do not
think the differences in the latitudinal ranges would have
occurred (MS 133) if there had been no disagreement about the
amount of the maximum declination, and if there had been no
confusion in evaluating the sines of degrees of a circle, because
9 the Byzantine and Indian methods are different. Also, if there
had been no corruption in the transmission of sine tables, which
corrupts any computation based on them.

First, I say that for political reasons and for fixing the
12 demarcation lines between kingdoms, the inhabited world was
partitioned into seven circular parts, like the six circles which
encircle (tangentially) a seventh circle, when all the circles are
equal. The reason for this division is that the great kings were
natives of Īrānshahr

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which consists of ᶜIrāq, Fārs, Jibāl, and Khurāsān. During the
dawn of creation and before the spread of mankind over various
regions, some of those great kings subjugated all these kingdoms.
3 Such kings had to come down on the central region to satisfy
their own ambitions and to take freely whatever they wanted.
Some did not rule over it, in particular, those who reigned after
Alexander. However, any one of these kings was regarded with
considerable reverential fear. His menace was warded off by
annual tributes, and by propitiation with gifts and gestures of
6 friendship. He was in need of a definite settlement of the demarcation lines between the kingdoms of others and his own.
He also needed a prevailing influence to get what he wanted, and
to instil in all those surrounding him a state of fear of his might
and a desire for his friendship. Those partitions were called
(MS 134) **keshvar,** which is derived from the Persian word
9 (**keshettah**) for line, to signify that each partition is distinguishable from the others, like what is bounded by lines. The first is
al-Wāsiṭa (i.e. the middle one) which is Īrānshahr, but they have
made it fourth, so that it really occupies a central position. Here
follows the figure (Fig. 23) showing their separate boundaries.

(MS 135) This partition has nothing to do with natural climatic conditions, nor with astronomical phenomena. It is made according to kingdoms which differ from one another for various reasons - different features of their peoples and different codes of morality and customs.

But people of the West, like the Greeks and others, who invariably adopted pragmatic and realistic methods in their work, contemplated the variations of natural phenomena along and across the parallels between east and west, and found no variations in the former directions other than a random distribution in the positions of the mountains, the seas, and the directions from which the winds blow. But when they studied the case

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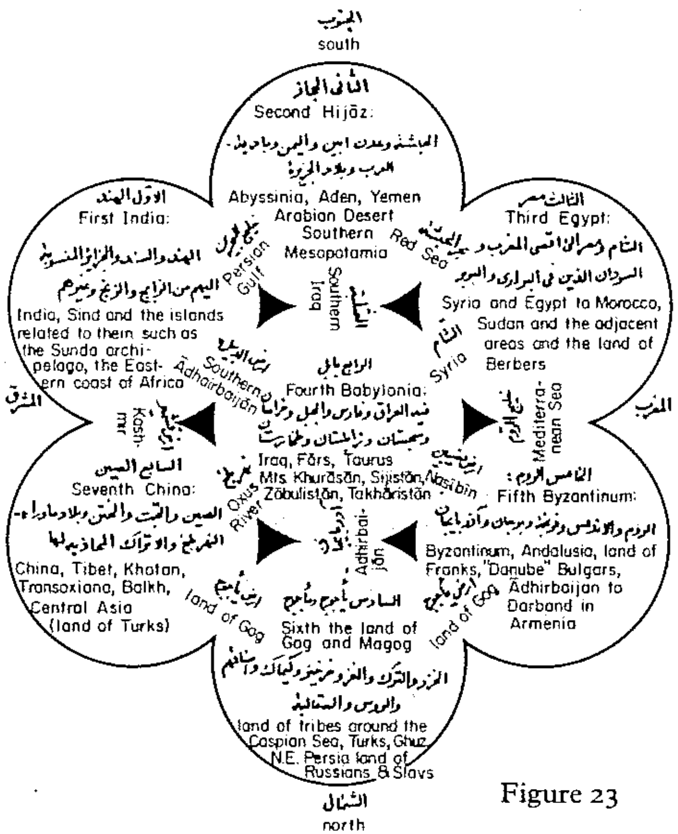

Figure 23

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of motion across the parallels towards the north pole or away from it, they found differences in the heat and cold of the air, and they found variations in zenith distances of the sun and the stars, in the altitudes of the pole and the surrounding stars, and in the folding up of the day by the night according to the displacement in that direction. So they partitioned the inhabited world into seven climates according to the most prominent variation, which is that between day and night, by parallel lines that extend from the eastern extremities of the inhabited world to its western extremities. They began with the middle of the first climate, and fixed it where the maximum (MS 136) summer daylight is of thirteen hours duration, and the middle of the second climate was fixed where the maximum daylight is thirteen and a half hours. Similarly, they proceeded and fixed the middles of the other climates by adding half an hour between each pair of consecutive middles, up to the middle of the seventh climate, where the maximum daylight is sixteen hours. Beyond that position, the land is thinly populated and the inhabitants are like savages. The farthest community lives in the town of Yūrah which can be reached from Ayswā in twelve days, and people travelling from Bulghār in wooden sleighs reach Ayswā in twenty days. They transport foodstuffs in them over surfaces of snow, and either drag them by hand or use their dogs to draw them. They also use sleds made of bones; they tie them to their feet, and with these sleds they can travel over long distances

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in short intervals of time. The people of Yūrah conduct the exchange of their merchandise by dumping their wares in one spot and standing away from it, because they are unsociable brutes; it is like the bartering of cloves on the high sea by people from the land of Lank.

The middle of the first climate was started where we mentioned because the habitation of civilized mankind begins there. The equator starts from the sea in (MS 137) the west, beyond the towns of the black people of the west, then it passes through sands and uninhabited lands near the sources of the Nile, then south of

Zanj beyond Nubia, then through Dībjāt and Wāqwāq islands, and the Zābij islands in the east. All those living below the equator are wild brutes who eat human beings, but among people living north of the equator, such wild and brutal dispositions and customs disappear gradually until the middle of the first climate is reached, where the people have become civilized and sociable, and have followed the rules of decent human conduct.

At this stage we ought to undertake the correction of the latitudes of the climates, and discuss the properties of regions of the earth according to their latitude coordinate. We consider it useful, if God permits!

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I say: If we suppose, in the figure given before (Fig. 22), that HK represents the maximum declination, and KE the maximum equation of daylight for each beginning, or middle, of one of the climates, the corresponding latitudes are worked out according to what was mentioned before about the derivation of the latitude when the arc of daylight is known. Also, let ABGD (Fig. 24) represent the meridian circle with center E, AE a segment of its line of intersection with the celestial equator, DZ a segment of its line of intersection with the declination circle, and BEG its line of intersection with the horizon. From T, the pole of the equator, we draw the line THE, then HE is the sine of (MS 138) the declination of the parallel, ZE is the sine of the rising amplitude, and HZ is the sine of the equation of daylight for that parallel of declination, I mean if HD is regarded as the total sine. But if we transform it to obtain the sine of degrees subtended by it in a great circle, the magnitude of HD will be given by the cosine of the declination of the parallel. Also, if we take the sine of the equation of daylight, it would be given by HZ, and its ratio to HD, the total sine, is as the ratio of HZ, the transform, to HD, the cosine of the declination of the parallel. Hence we multiply the sine of the equation of daylight by the cosine of the declination of the parallel. We retain this product, but we divide it by the total sine, in order to obtain the transformed HZ. Now ZE is the hypotenuse of a right triangle having it (HZ) and HE as legs, and its (ZE's) ratio to

HZ is as the ratio of ET to TK, the sine of TG. Hence we square both the transformed HZ and HE and divide the retained product by the (square) root of their sum. There results the sine of the latitude of the city and the climate. But HE, for all climates, is the same for the longest day, because it is the sine of the maximum declination. It is

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24;0,57,38, and its square is 7,474,985,764 fourths. Similarly, HD, the cosine of the maximum declination, to the same scale (?), is [5] 4;59,59,5. I prefer the latter operation, because it involves referring to sine tables only for the determination of one sine and one arc sine. Further, the approximation obtainable by using irrational (MS 139) roots is less (precise ?) than that obtainable by using sines, because the latter is a simple, single operation, but the former is a compound, multiple (operation).

If the partition of the climates according to hours of maximum daylight involves a difference of half an hour between any two consecutive middles, then it is known that their beginnings and ends differ from the respective middles by a quarter of an hour, so that the difference between any two consecutive beginnings shall be half an hour, and similarly for the ends. I have computed them, according to what I have stated, and placed the results in this table (which follows). (Page 140 is blank in the MS).

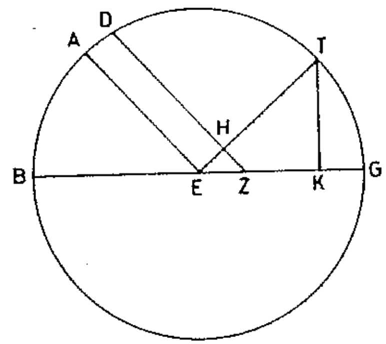

Figure 24

(MS 141)

Table of Climates

THE CLIMATES Beginnings and Middles The end of a climate is the beginning of the next.		Maximum Daylight		Equation of Daylight			Sine of Equation of Daylight			Sine of Equation of Daylight Transformed			Sine of Rising Amplitude			Sine of Latitude			Latitude		
		Hours	Minutes	Parts	Minutes	Seconds	Parts	Minutes	Seconds	Parts	Minutes	Seconds	Parts	Minutes	Seconds	Parts	Minutes	Seconds	Parts	Minutes	Seconds
First Climate	Beginning	12	45	5	37	30	5	52	52	5	23	28	24	36	39	13	8	13	12	39	5
	Middle	13	0	7	30	0	7	49	54	7	10	44	25	3	58	17	11	3	16	38	34
Second Climate	Beginning	13	15	9	22	30	9	46	25	8	57	33	25	37	58	20	58	16	20	27	29
	Middle	13	30	11	15	0	11	42	20	10	43	48	26	18	13	24	28	33	24	4	30
Third Climate	Beginning	13	45	13	7	30	13	36	58	12	28	53	27	3	58	27	40	8	27	27	40
	Middle	14	0	15	0	0	15	31	45	14	14	6	27	52	2	30	35	39	30	39	27
Fourth Climate	Beginning	14	15	16	52	30	17	25	1	15	57	56	28	50	19	33	13	1	39	36	56
	Middle	14	30	18	45	0	19	17	11	17	40	45	29	49	18	35	34	11	36	21	29
Fifth Climate	Beginning	14	45	20	37	30	21	8	6	19	22	25	30	51	23	37	40	20	38	53	36
	Middle	15	0	22	30	0	22	57	40	21	2	52	31	56	3	39	32	45	41	13	52
Sixth Climate	Beginning	15	15	24	22	30	24	45	44	22	41	55	33	2	44	41	12	49	43	23	5
	Middle	15	30	26	15	0	26	32	14	25	19	33	34	11	1	42	41	55	45	22	8
Seventh Climate	Beginning	15	45	28	7	30	28	17	1	25	55	36	35	20	27	44	1	1	47	11	26
	Middle	16	0	30	0	0	30	0	0	27	30	0	36	30	31	45	11	41	48	52	21
	End	16	15	31	52	30	31	41	2	29	2	37	37	41	13	46	15	13	50	24	34

142 (MS 143)

The inhabited world is not discontinuous beyond the end of the seventh climate, nor before the beginning of the first. However, habitations become less and are found in certain regions and not in others, because the heat south of the first climate scorches, except in places situated near mountains or seas. The desert lands of the Sudān are scorched and can not produce any vegetation which is essential for the growth of the animal genus, and there is no temperate air for breathing, which is of basic importance for its existence. On the other hand, there are habitations on the adjacent islands, and if the inhabitants are not reckoned as ordinary people, (nevertheless) it is permissible to classify them under the human species.

Similarly, the cold is destructive north of the seventh climate. Its biting severity, its long duration, and the accumulation of snow, which never melts or if it does, then only for a short time, prevent the cultivation of fodder for animals, except in a region situated under favorable conditions.

Thus we see that the northern regions are not habitable because of cold and snow. However, there are people who inhabit the coast of the sea which turns away from the Circumambient Ocean to the north of the Slavs (**Ṣaqāliba**); and it is known as the Sea of the (Waranj), Varang because this nation lives on its coast, in places adjacent to those frozen regions but not so severely cold. We even find among those people some who go out on that sea in the summer, for fishing (MS 144) or piracy, and they proceed in the direction of the north pole to the region where the sun revolves above the horizon at the summer solstice. On seeing it, they feel proud among their fellows because they have reached the place where there is no night.

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The suppression of habitations in both east and west directions is not due to excessive heat or cold, because these factors are not operative. It is due to the fact that the world was evolved, as was mentioned before, from a totality of water according to a divine plan, and not as a consequence of the

interaction of natural forces. The plan was to evolve a selected
part and to exclude the rest, so that the waters shall surround it.
Hence there is necessarily an extremity of the part on the eastern
side as well as the western side.

6 The sea on the south side of the world, I mean that which
issues forth from the ocean east of China, extends along the
equator and spreads out to the coasts of China, India, Fārs,
Arabia, and finally ends up in a narrow strait at the Red Sea
9 (Al-Qulzum); and it is called, at any given place, by the name of
the country adjacent to it. Also, the sea which issues forth from
the ocean west of Zanj and at the promontory called Barāsūn (?),
extends similarly to the south of the equator, and spreads out
along the coasts of the Sudān and south of Zanj. Both seas fall
directly under the influence of the sun, moon, and the planets,
12 and hence (MS 145) their airs are soft, and navigation in their
waters is smooth.

But the ocean in the west, where most of the water lies,
has many shallow parts, and in most cases its depth is small and
15 its water is thick. It is the muddy source where navigation is
impossible, because its lanes are not known. This is why Hercules
the Giant constructed his signposts and pillars near Andalūsia
to prevent sailors from proceeding beyond them. They were
erected on land, but the water later submerged them for the
18 reasons which were mentioned above, or for some similar reasons.

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An erudite gentleman reported in a letter to Ḥamza bin al-
Ḥasan al-Iṣbahānī about the wonders he had seen in the west.
3 He stated in it that he had crossed the narrow lane in a ship. This
is the strait that connects the Sea of Syria (the Mediterranean)
with the ocean, and from which the two shores are visible, that of
Andalūsia, and that of the districts of Tangier and farthest Sūs.
On looking down into the water, he saw a bridge made of a series
6 of arches built of stone, well below the surface. Some of those
on board claimed that it was built by Alexander, but the Anda-
lūsians said «Woe be to Alexander! Did he rule over our land so
that he could do that? It is surely the work of the ancient
Hercules». I do not think that the bridge of Hercules, mentioned

9 in the book (called) the Geography, is anything else but this, and
there is (MS 146) no doubt that the bridge was above the water,
because it was constructed for crossing over it, but it was later
submerged after the water had risen.

As to the ocean in the east, it is very dark and without
12 current, and there is great danger in travelling on it. It is thought
that these two seas, west and east of the inhabited world, are
separate seas, but travellers on both of them, whose ships were
wrecked by storms, have told stories which make people assume
that they meet. Also, there is evidence in our time to support this
assumption, and even to prove its validity, because stitched ship
15 planks have been found in the ocean near its juncture with the
Mediterranean. These are made for the Indian Sea,because there
is an abundance of lodestone in it which is not found in the
western sea,where ships that sail on it have their planks attached
with iron nails but are not stitched. The presence of those planks
18 on it is a proof that they have reached it through a juncture
between them. This can not be through the Red Sea because
there is an isthmus between them.

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Also, it is unlikely that their juncture is through the sea in the
north because those planks, which were smashed in the Indian
Sea, would have to move out of it through the juncture at the
3 eastern strait, then turn round in the direction of the North
Pole or around the opposite northern quarter which is relatively
south of the inhabited quarter. (MS 147) But not all that is
possible takes place. It is more plausible to imagine that their
juncture is south of the inhabited world. In particular, those
6 who talk about their juncture have mentioned that the eastern
water is at a higher level than the western one, like what was
found by a geological survey about the higher level of the water
of the Red Sea relative to the Mediterranean. It is possible
that this higher level is due, among other causes, to the flow
9 of the tide from east to west under the influence of the moon.
I shall discuss these causes in a separate book on the subject
of the tides, if God, Most Munificent, supports me to do it.

After that digression, I resume my discussion and say that

beyond the place I defined as the end of the seventh climate and up to the place whose latitude is the complement of the maximum declination, the maximum daylight increases gradually until it becomes twenty-four hours. The amounts of variation that take place in the duration of maximum daylight take place equally in the duration of maximum night at the winter solstice. If a traveller proceeds northwards beyond that place, then the sun, as seen by him, would rotate over the earth as long as its declination is greater than the colatitude of his locality, and the duration of that rotation is reckoned as one day. To find its amount, we take the arc of the colatitude as an entry of declination in the declination table and find

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the corresponding (MS 148) (longitude) entry, which is (tabulated in) equal steps. The latter entry is the displacement of the part (i.e. the longitude of the sun), measured from the vernal equinox, for the beginning of the longest day. We subtract that displacement from a hundred and eighty, and there remains the distance of the part which marks the end of the longest daylight. Then the longitudes of the mean sun are found from these two longitudes, with reference to the apogee which has been corrected for the given time. The duration of the mean motion between those two positions occupied by the sun is the amount of the longest day there. Its amount continues its increase as the journey proceeds farther (north) so that the daylight takes from the spring and summer quarters, and the corresponding night takes from the nights of the autumn and winter quarters until his hypothetical, not actual, penetration finally ends up at the north pole. There the whole year becomes one daylight and one night, by the rotation of the celestial sphere like a mill. We discussed previously the determination of latitudes of places from the data of maximum daylight and the solar longitude, so there is no need to do it again for these places.

We still have a computational method, devised by Muḥammad bin al-Ṣabbāḥ, for the determination of the maximum rising amplitude from the observations of three rising amplitudes made at the extremities of two equal time intervals. He described

the method in his treatise without giving a proof, and it is a (MS 149) good method, although it is based on oversimplification. I shall here reproduce it as he presented it in his treatise, but the proof of its validity will become clear when I apply it to some of my own observations.

Here is what he stated in his treatise: We measure the rising amplitude at sunrise with an alidade, and we retain twice

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its sine, as the «first (number)». We also measure the rising amplitude after a lapse of one month, approximately, and we retain twice its sine, as the «second (number)». Then we measure it after a lapse of a second interval, equal to that same interval, with the proviso that both intervals occur in the same season, and we retain twice its sine, as the «third (number)». Then we multiply the first retained (number) by the third retained (number), and subtract the product from the square of the second retained (number). We call the square root of the remainder the «extracted chord». Then we add the first and third retained (numbers), and square half the sum obtained. We deduct this square from the square of the second retained (number), and we call the square root of the difference of the two squares the «perpendicular». Then we multiply the extracted chord by the second retained (number), and we divide the product by the perpendicular. The quotient obtained thereby is the maximum rising amplitude.

I discussed above the determination of the declination of the part from the rising amplitude, the latitude of the city being known. The author of this operation observed (MS 150) the rising amplitude at the extremities of two equal intervals in order to obtain two equal arcs on the rising amplitude circle. This would have been the case if the motion of the sun were uniform during the two intervals, but the observed apparent motion is not uniform, and so the equality of the arcs for those two intervals does not hold, unless the sun is at apogee, or perigee, in the second observation. However, this difference may become imperceptible, if the time interval is diminished, but this is unfavorable to an accurate determination of the rising

18 amplitude, which requires a big difference in the amplitude with
little difference in the sun's velocity, but that is not available
in this period of time. Since the declinations of the degrees
(of longitude) are equal to the rising amplitudes at the terrestrial
equator, and the meridian circle of any locality is one of its
(the equator's) horizons, we use the declination at it, because we
21 have transformed our observation to it (the equator).

Let us present a discussion of the work before we give the
example. Let ABG (Fig. 25) represent the ecliptic, with

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center E. From E, we erect EB perpendicular to the diameter AG.
We take (arc) GD equal to the maximum declination, and we
3 draw DZ parallel to GE. With center E and radius EZ, we draw
the circle EH (ZMH), which is the declination circle. We suppose
AT to be an arc of the ecliptic, and we join EST. We drop SO
and TF perpendicular to AG; then (MS 151) we draw SC
6 parallel to EA. AC is the declination of arc AT because, in the
two triangles ETF and ESO, the ratio of TF to SO is as the ratio
of TE to SE. But SE is the sine of maximum declination, TF the
9 sine of the given arc measured from the (vernal) equinox A, and
ET is the total sine. Therefore SO is the sine of the declination of
arc AT, because it is explained in elementary astronomy that the
ratio of the sine of a given (ecliptic) arc to the sine of its decli-
nation is as the ratio of the total sine to the sine of the maximum
12 declination. Now, SO represents the sine of arc SH, on the
supposition that EH represents the sine of maximum declination,
ZSH the maximum declination, and SH the declination of AC.
But we require the declination in a circle whose circumference is
15 three hundred and sixty parts, so it is either the arc AC, or we
transform SO to parts of the total sine in the great circle. We
say that the ratio of the radius ES, representing the sine of
maximum declination, to SO, measured in this scale, is as the
ratio of ES, representing the total sine, to SO, measured in that
18 scale. This is the transformation required for the problem just
mentioned. Then we take the two arcs, AY and AK, such that
the two arc differences, TY and YK, are equal (i.e. AY - AT =
AK - AY). We join EMY and ELK; then the two arc differences

SM and ML will also be equal, because the arcs are similar. The
21 inaccuracy in this argument is his assumption that arcs TY and
YK (MS 152) are equal because of the equality of the two time
intervals between the two observations.

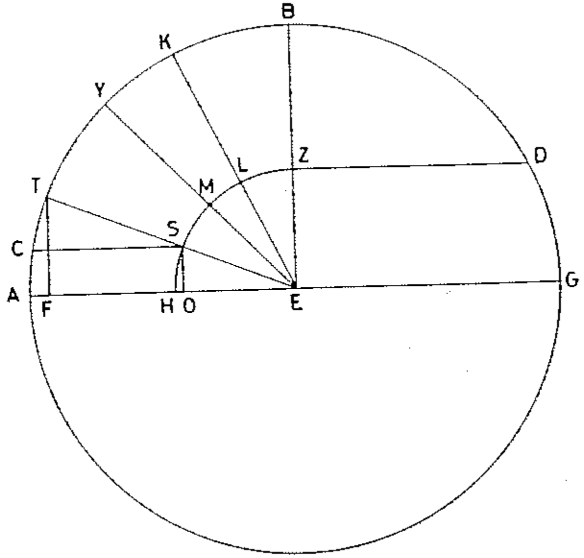

Figure 25

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After that introduction, we return to his operation and
illustrate it with three successive meridian observations, at an
3 interval equal to the days of one month. The first is the meridian
altitude observed on Wednesday, the third of Ṣafar, year four
hundred seven of the Hijra, (being) Bahman day, the second of
the month Murdād, year three hundred eighty-five of Yazdigerd;
I found it at Khwārizm to be 69;11°, and since I had found the
6 latitude to be 42;17°, the declination must be 21;28°.

The second is the meridian altitude observed on (MS 153)
Friday, Bahman day, the second of the month Shahrīvār of

that year; it was found to be 61;43°. So the declination must be 14;0°.

The third is the meridian altitude observed on Sunday, Bahman day, the second of the month

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Mihr of that year; it was found to be 50;55°. So the declination must be 3;12°. Let us call this third one «the first», and the first one «the third». This is not necessary, but it is convenient to call the one nearest to the equinoctial point the first.

Let ABGD (Fig. 26) be the declination circle, and let us suppose that point A on it corresponds to the equinoctial point. We take arc AB on it equal to the first declination, which is 3;12°; AG equal to the second declination, which is 14;0°; AD equal to the third declination, which is 21;28°. (Also), we take arc AE equal to arc AB, and arc DZ equal to arc DE, and we draw BE, BD, BZ, and DZ. We (also) drop DH perpendicular to BZ. (We note that) chord BE is twice the sine of the first declination. It is the first retained (number) which is 6;41, [55]. (Also,) twice the sine of AG is equal to the chord of DZ, because DZ equals DE, and AG is half EBD, which is equal to DZ. So the chord of DZ is 29;1,50, which is the second retained (number). Similarly, the chord of BZ equals twice (the chord of) AD, because if we draw DM parallel to BZ, then arc MZ will be equal to the arc DB, and the arc MD will be equal to arc BE. (MS 154) So arc BDZ will be equal to twice DB and twice BA. And half the sum of these two doubles is arc AD. So the chord of BZ is 43;54,55, which is the third retained (number).

The line ZBE (ZBD) is a broken line in this circle, so if we add to it MZ and MD, an inscribed quadrilateral MZBD is formed, and according to

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the first treatise in the Almagest, the product of the two diagonals, MB and ZD, is equal to the product of MZ times DB plus the product of ZB times MD. But ZD and MB are equal, and likewise MZ and DB are equal, and MD and BE are equal. So the square of ZD is equal to the square of DB plus the product of ZB

times BE, and because the square of ZD equals the sum of the squares of ZH and HD, and the square of DB equals the sum of the squares of BH and HD, hence the sum of the squares of ZH and HD equals the sum of the squares of BH and HD plus the product of ZB times BE. Now the square of DH is common between the two sides. So if we subtract it there remains the square of ZH equal to the square of HB plus the product of ZB times BE. So ZBE is a broken line bisected at H, but at B (the broken segment is) divided into unequal parts. Hence ZH is equal to the sum of HB and BE. So if we multiply BE, the first retained, by BZ, the third retained (number), (MS 155) the product is 3,813,460,925 fourths. If we subtract this from the square of DZ, the second retained, which is 10,940,340,100 fourths, there remains 7,127,879,175 fourths, which is the square of BD. Hence its root, which is 84,427 seconds, is DB, the extracted chord. Since H is the midpoint of the broken line ZBE, and BE plus BZ is the sum of the first and third retained, hence ZH, which is half their sum, equals the sum of their halves. Therefore it is equal to the sum of the sine of AB, the first declination, plus the sine of AD, the third declination, that (sum) being 25;18,25. Its square is 8,300,121,025 fourths. So if we subtract it from the square of DZ, the second retained, there remains the square of D [H], 2,640,219,075 fourths. Its root, the perpendicular, is 51,383 seconds.

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We draw in the circle the diameter DT, and we join ZT. The two angles DBH (and) ZTD are equal because both are subtended by the same arc ZD; and the two angles TZD (and) DHB are right (angles). Therefore the triangles TZD (and) DHB are similar, and the ratio of BD to DH is as the ratio of TD to DZ. So if we multiply BD, the first, it being the extracted chord, by DZ, the fourth, it being the second retained, the product is 8,823,465,770 fourths. And if we divide that by DH the second (part), which is the perpendicular, the quotient obtained thereby is TD, the third (part), as 47;42. (MS 156) Half of it is 23;51, which is the sine of the maximum declination, and its arc (sine) is 23;25,19°. The discrepancy between it and the established

amount is intolerable. It is due to two causes: first, the frequent use of sines and roots, and second, the assumed equality between arc BG and arc GD, on account of the equality of the two time intervals. That equality can not hold unless the middle observation is made when the sun is exactly in apogee, or in perigee. But this is not possible in our time because both are near the solstices, so it is impossible to have, during one season, two equal arcs on both sides of either apse, because the difference in declination at the extremities of the arcs would be too big. (MS 157)

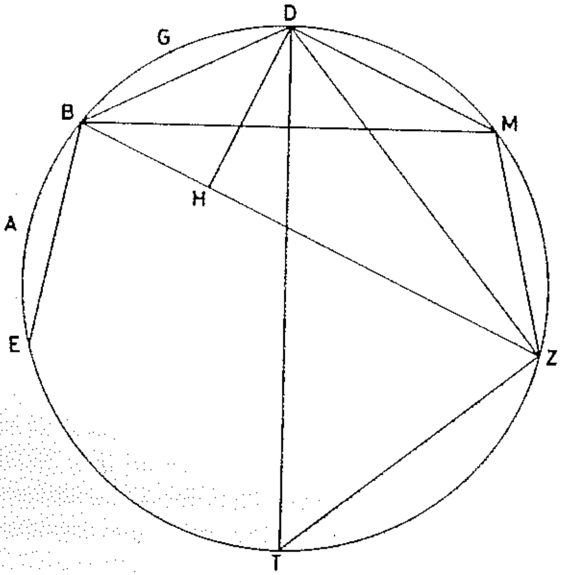

Figure 26

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Muḥammad bin al-Ṣabbaḥ had another method which was corrupt in the manuscript copy of his treatise that has fallen into my possession. However, Abū Naṣr Manṣūr bin ʿAlī bin ʿIrāq has worked out a method which is either the same as the original

method, or is an independent third method. It is as follows. He said in **al-Majisṭi al-Shāhī:** We observe the rising amplitude of the sun, and we retain twice its sine as the first (reference number). We wait as long as we please, provided the interval is in the same season, then we observe again its rising amplitude, and we retain twice its sine as the second (reference number). We add the two retained (numbers), and we multiply half the sum by the total sine. Then we divide what was obtained by the cosine of the sun's displacement along the ecliptic in (the interval) between the two observations. We multiply what was obtained by itself, and we subtract from it the product of one of the two retained (numbers) by the other. We take the (square) root of what was obtained, and we multiply it by twice the total sine. Then we divide the product by twice the sine of the sun's displacement along the ecliptic, (in the interval) between the two observations. The quotient obtained thereby is the diameter of the circle of maximum rising amplitude.

As in the last example, let AB represent the first rising amplitude, and BG the second rising amplitude. We draw BE, the chord of twice AB; so it will be the first retained (number). And BZ, the chord of twice BG, will be the second retained (number). (MS 158)

But, for illustration, let AB (Fig. 27) represent the first of the three declinations which I observed. It is 3;12°, and so (chord) BE is 6;41,55. Let BG represent the second declination, which is 14;0°, then chord BZ is 29;1,50. We bisect (the arc) EBZ at D, and we drop DH perpendicular to BZ. So ZH will be half the sum, 17;51,32. And because DG equals AB, hence it (AD)

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will equal BG, and BD is the difference between the two rising amplitudes. Its ratio to a quadrant of this circle is as the ratio of the sun's apparent displacement, in the interval between the two observations, to a quadrant of the ecliptic. This interval is of thirty days duration, uncorrected by the equation of time, and the sun's displacement in it, according to Ḥabash's zīj is 29;17°. The complement of this is 60;43° and the sine of this complement is 52;19,57. Let O be the center of this circle, and

6 let us draw OD and OB. (Angle) DOB is a measure of the sun's displacement in the interval between the two observations. We bisect angle DOB by line OF; hence angle DOF is a measure of half that displacement, and angle ODF is a measure of the complement of that half.

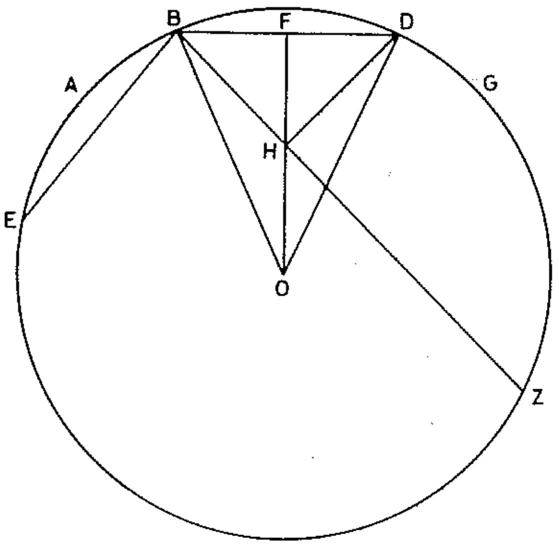

Figure 27

9 But angle DOF is on half the arc which subtends angle DZB, so the two (angles) are equal, and the two triangles DOF (and) DZH, right angled at F and H, respectively, are similar. Therefore angle DZH in the circumcircle of (MS 159) triangle DHZ, is
12 (equal to) 14;38,30°, I mean half the sun's displacement. And the angle ZDH, the complement of that, is 75;21,30°. Its sine is 58;3,5. And the ratio of HZ to ZD is as the ratio of the sine of
15 angle ZDH to the sine of angle DHZ, a right angle. So if we multiply HZ, half the sum of the retained (numbers), by the total sine, we get as product 3,857,520 seconds. And if we divide it by the sine of angle ZDH, the quotient obtained is 18;25,55. That is

DZ, and its square is 4,402,986,025 fourths. Because the square
18 of DZ equals the sum of the square of DB plus the product of ZB times BE, we subtract (from the square of DZ) the product of one of the two retained by the other, which is 2,520,258,650 fourths. The remained is 1,882,727,375 fourths and its (square) root is 43,390 seconds, which is BD. Now the ratio of its half,
21 which is DF, to DO, the radius of the circle, is as the ratio of DF, representing the sine of half the sun's displacement, to DO, representing the total sine. So if we multiply DF by the total sine, the product is 1,301,700 seconds, and if we divide by

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the sine of half the sun's displacement, it being 15;9,59, the quotient is 23;50,28, which is DO, the radius of the circle. Its
3 arc (sine) is 23;24,46°, the maximum declination. This result is approximately equal to that derived by the first method. (MS 160)

I have discussed sufficiently what I intended to say about the interrelationship between, and the determination of, latitudes of cities, maximum declination, partial declination, and associated problems from known data of meridian altitudes, altitudes of
6 known azimuths, rising amplitudes, and arcs of daylight. Having dealt with latitude, I shall presently deal with the problem of longitude.

V. ON THE DETERMINATION OF LONGITUDINAL DIFFERENCES BETWEEN CITIES

As latitude is measured towards a point that actually exists, from a circle (MS 161) which exists relative to that point, it has a determinate beginning and an end. But since longitude is measured along that circle or parallel to it, and as the circle is a continuous curved line that has no actual point (of beginning) except a hypothetical one, or relative to something else other than that circle, longitude has no actual beginning or end. However, as the inhabited world does not spread over a complete circuit, there exist two extremities in longitude, an eastern extremity and a western one. Further, according to scientists interested in this matter, the two extremities are approximately on one of those circles that pass through the two poles, and hence the inhabited world is extended over half a circuit of the earth. It happens that this fortuitous occurrence is all the better, because the lesser of the two geodetic coordinates is then rightly called breadth (latitude), and the bigger one length (longitude).

People on both sides have measured longitude from their extremity of the inhabited world. The Chinese, Indians, and Persians measured it from the eastern extremity, but the Byzantines, Greeks, and Egyptians measured it from the western one, at the five islands in the Circumambient Sea known as the Ocean. The islands are near the land of the West (Morocco) and are called (those of the) Immortals, or the Happy Isles, and though they are at a distance of two hundred farsakhs from the coast, they are regarded as the initial inhabited locality.

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Ptolemy measured longitude from them, and made the longitude of the eastern extremity one hundred and eighty degrees. (MS 162)

The easterners followed the same principle, extending the range of longitude for the inhabited world over half a circuit, but locating the initial meridian on their side. They made the range of longitude half a revolution by demanding that, if one and the same lunar eclipse occurs west of the eastern extremity, it would also occur east of the western extremity, and that the interval between rising and setting shall be approximately twelve hours.

When a comparison was made between the two systems, it was found that the longitude of one and the same place, as determined by the easterners, exceeds the supplement of its longitude, as determined by the westerners, by ten degrees. Al-Fazārī assumed in his zīj that the difference is thirteen and a half degrees. If he has taken the initial meridian through the Immortal Isles, then the final meridian should have been taken beyond the extremity adopted in the east by that said amount, and if he has taken as initial meridian that one which is adopted in the east, then the final meridian through the shore of the sea in the west should have been taken beyond those isles. It is for this reason that the recorded longitudes for any one city are different. For example, some give the longitude of Baghdad as seventy degrees, while others give it as eighty degrees.

This is the concept of longitude in principle. We need it (longitude) first, for a cartography of the earth. (MS 163) He who has knowledge with insight will not be disappointed with what I have said concerning the different points of view about the initial and final extremities attributed to the inhabited world, and his work will not suffer,

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if he does not disregard a critical contemplation of it. But if he acquires knowledge in the traditional manner and does not fully comprehend the circumstances which have led to divergent views and the tabulation of conflicting results in one table, his operations, in particular those involving eclipses and to a greater extent solar eclipses, will suffer from obvious confusion. What one really needs concerning longitudes is a knowledge of the longitudinal difference between different towns. If we obtain that, we do not need these final and initial meridians. We may

even be able to correct their positions, if the Lord of Time helps
me to do so, as he helped Ptolemy and former scientists who
were interested in this matter. However, the hope for such an
achievement can hardly be realized, because I have explained
my circumstances in what preceded.
9 By the difference in longitude between cities, I mean the
arc which is intercepted on the equator between their meridian
circles, or that intercepted on any one of the parallel and similarly
situated circles. We have known from the principles of astronomy
12 that there is no difference in longitude between any two cities
whose zeniths fall on one meridian circle, and that midday
in both of them takes place (MS 164) simultaneously. The
(times of) sunrise and sunset in both of them are identical when
the (sun's) daily path coincides with the celestial equator, but
15 they are different when the daily paths are displaced from it.
If the day-circle is a northern one, the time of sunrise at the
city of higher latitude, is earlier than sunrise at the other but
the time of setting is later, and if it is a southern day-circle,
then sunrise at the city of higher latitude is later than the other,
but sunset is earlier.
18 Moreover, any two cities situated on one and the same
parallel have no difference

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in latitude, and the arc between their meridians is a measure of
their difference in longitude; and the difference between them
in rising and in setting in that parallel is invariable for the same
difference in longitude.
3 Also, any two cities which are neither on the same meridian
circle nor on a single parallel of latitude have different longitudes
and different latitudes, and the arc between their meridians is a
measure of their difference in longitude. However, the difference
6 in rising and in setting is compounded of both matters.
Therefore, for any two cities, there are three cases to consider.
First, when both have the same latitude but different
9 longitudes. Second, when both have the same longitude but
different latitudes. Third, when the two (coordinates for each
of them) are different.

It is impossible to have coincidence (identical coordinates)
between them, and this is particularly revealed by a critical
enquiry but not by empirical evidence. (MS 165) (Either) the
latitudes of any two points on the earth, or (else) their longitudes,
12 must differ, but instruments can not detect that difference, if
it is of a small order. There is no objection to looking at this
in a figure under our sight, because the mind progresses from a
concrete example to conceive of a reasonable abstraction.
15 For the first (second) case, let ABGD (Fig. 28) represent
the meridian, AEG half the equator, AT the latitude of a city
whose horizon is BED, AK the latitude of a city farther north
18 than T, whose horizon is ZEH. We suppose LMS to be a parallel
of northern declination, then it is known that a (sun) rise at
it from the horizon ZEH, at a point S, is prior to a (sun) rise
at point M on horizon BED by the amount SM, which is the
difference between the two halves of daylight, for this parallel
in both cities.

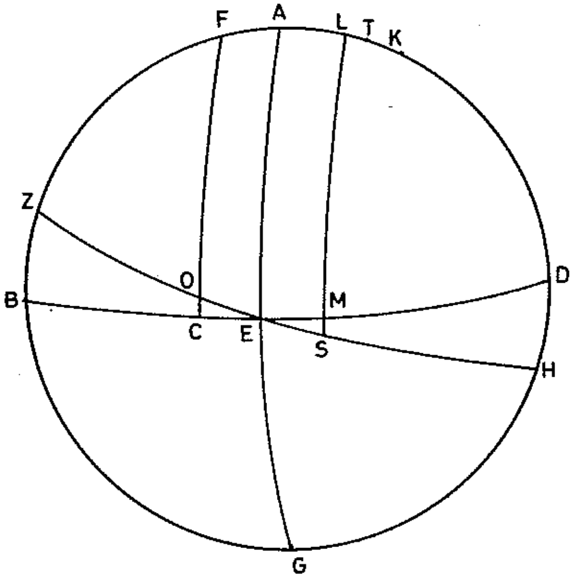

Figure 28

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Further, we suppose FOC to represent a parallel south of the equator. Then it is obvious that (sun) rise at it, on horizon ZEH, takes place at point O after the (sun) rise at point C on horizon BED. This is the opposite of what was inferred in the case of a northern parallel. Also, OC is the difference between the two halves of daylight for this parallel in both cities. But (sun) rise at the equator takes place at point E, which is common to both horizons. Since the rising point of the equinox is the pole of the meridian common to both cities, there is only one rising point, and that is just what we stated. (MS 166)

For the second case, let ABGD (ABG) (Fig. 29) represent the horizon, AEG the meridian, E the zenith, DBC an arc of the equator whose

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north pole is Y and south pole is W, and ES an arc of the parallel whose declination DE is equal to the latitude of the city. Around the two poles Y and W we construct the two parallels AR and ΘG, which are tangential to the horizon. We suppose that point T, on the parallel ES, is the zenith of another city, and we draw a great circle through the points W, T, and Y. This circle is the meridian through T, and YΘ is part of it. With T as pole and with distance equal to the side of the square, we draw a (great) semicircle RMΘ. Each of (the arcs) TK, YΘ, WT, and the latitude of T, is equal to DE. The difference in longitude between the two cities is (MS 167) that (arc) which is between their meridians. If it is measured on the equator, it is DK, but on the parallel ES, it is given by ET, and ET is similar to DK. The interval between risings in the two cities on the parallel ES is HS, and this is equal to ET. To prove this, let us draw from Y to the equator the two arcs YHL and YSN. It is obvious that the equation of daylight for one parallel is the same for all (other) places on the same parallel of latitude, and hence arc BL is equal to arc MN. Since each of the two arcs DB and KM is a quadrant, arc DL is equal to arc KN. So if we subtract the common arc KL, there remains arc DK equal to arc LN. But HS is similar to LN, and ET is similar to DK, therefore HS is equal to ET.

Further, we suppose that rising is taking place in a parallel north of the parallel ES. Let

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OF be the arc of that parallel which is intercepted between the two horizons, and let us draw from pole Y to the equator the two arcs YON and YEC. Since arc BN is equal to arc NC, DN is equal to KC. But KN is common to both, and if we subtract it, there remains NC equal to DK. Since OF is similar to NC, the difference in risings on this parallel is also an arc which is similar to the arc between the two longitudes. The equatorial arcs DB and KM are both quadrants, and arc KB is common to both, therefore arc BM is equal to arc DK. Next, we suppose that QX (MS 168) between the horizons, from a is the arc intercepted parallel south of the parallel ES, and we draw from pole Y to the two points Q and X, respectively, the two arcs YZQ and YLX.

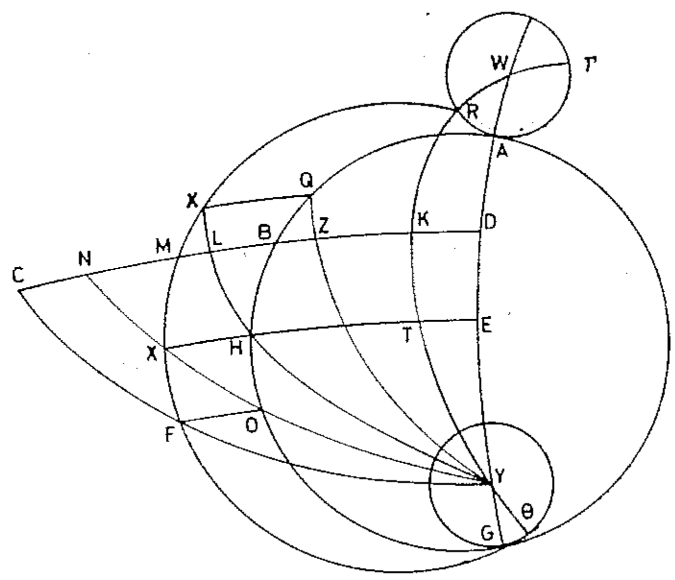

Figure 29

Since arc ZB is equal to arc LM, arc DZ is equal to arc KL, and since KZ is common to both, there remains arc ZL equal to arc DK. But QX is similar to ZL, therefore ET and QX are similar. Hence the difference in rising and in setting in two cities of equal latitudes, is of the same amount as the difference between their meridians. (MS 169).

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For the third case, we reproduce (the part) of this figure (Fig. 30) which we need, and we suppose that T is not on [north of] the parallel ES. Thus TK, the latitude of T, is greater than DE, the latitude of E, and therefore Θ is not on the circle (through the point of contact) G because YΘ, which is equal to TK, is greater than YG which is equal to DE.

From pole Y we draw the arcs which define the equations of daylight. Thus LB is the equation of daylight for the declination HL, at latitude YG. So the ratio of the sine of BL to the total sine is as the ratio of the reversed shadow (**or** tangent) of LH to the reversed shadow of the complement of YG. Also, MN is the equation of daylight for the declination SN. So the ratio of the sine of MN to the total sine is as the ratio of the reversed shadow of NS to the reversed shadow of the complement of YΘ. To put these disturbed ratios in order, we say that the ratio of the sine of BL, the first, to the tangent of LH, the second, is as the ratio of the total sine, the fifth, to the cotangent of YG, the sixth. Also, the ratio of the tangent of NS, which is equal to the tangent of LH, the second, to the sine of MN, the third, is as the ratio of the cotangent of YΘ, the fourth, to the total sine, the fifth. Hence, from equalities in the confused ratio, the ratio of the sine of BL to the sine of MN is as the ratio of the cotangent of YΘ to the cotangent of YG. But the complement of YΘ is less than

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the complement of YG, and this relation holds between the tangents of the complements. Therefore the sine of (MS 170) BL is less than the sine of MN and consequently arc BL is less than arc MN. If the arcs were equal, arc LN would have been equal to

arc DK, and arc HS would have been similar to arc DK. But since they are different, that similarity no longer holds. However, DL is half the daylight (length) for the declination LH at the horizon of city E, and KN is half its daylight at the horizon of city T. So the difference between them, which is LN, is the difference in risings for the parallel ES. Similarly, it can be seen that BQ, the equation of daylight for the declination QO at the horizon of city E, is not equal to MC, the equation of daylight for the declination CF, at the horizon of city T. The difference in rising (times), OF, is similar to QC, which is equal to the difference between the two halves of daylight, DQ and KC. (MS 171)

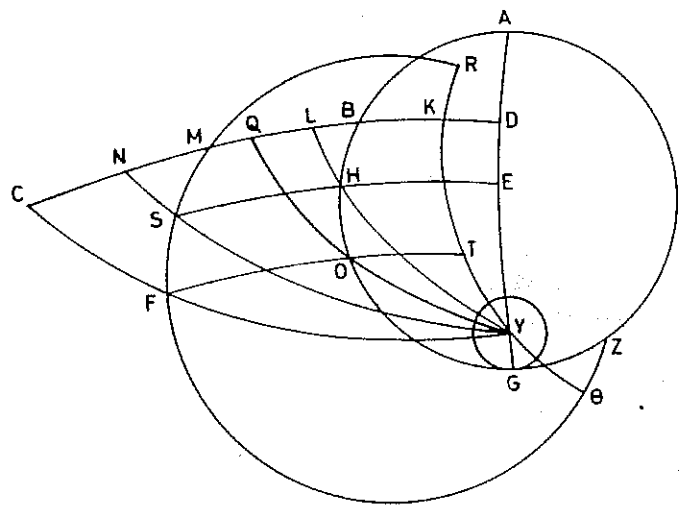

Figure 30

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In all three cases, if rising and setting coincide with the two points of intersection of the two horizons, they occur in cities simultaneously. In the first case, they are the rising and setting points of the equinox, but in the latter two cases, both

are displaced from the east-west line and their azimuths are at a distance from it.

I am deferring the discussion of these two points until I have dealt with the determination of longitude, because they can not be determined without a knowledge of both the latitude and the longitude. It is known that if the rising takes place on arc LM, it occurs earlier at city T, east of city H, but if it (T) happens to be on the arc WR then it occurs later at city T than at city E.

Such an occurrence can hardly be imagined, except by one who can truly visualize astronomical phenomena, and there are unexpected occurrences like it which are readily denied by one who has not disciplined himself to provide a proof for his refutation. For example, when the sun is at twenty-four degrees in the sign of Aquarius, its latitude is forty-two degrees east, in a locality of latitude thirty-six degrees, and its ascendant is nine degrees in the sign of Gemini. But when it is twenty-three degrees in the sign of Pisces and its altitude is also forty-two degrees east, its ascendant is also nine degrees in Gemini. One who does not know that, readily imagines that there should have been a progression in the position of the ascendant (MS 172) along the signs which is approximately equal to the advance of the sun from its first position to the latter one. Abū Naṣr Manṣūr bin ʿAlī bin ʿIrāq has sent me a comprehensive treatise (**risāla**) in this connection.

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Now I say: If we want to determine the longitudinal difference between one city and another, we have to ascertain the same moment of time itself in both of them. Since the beginnings of daylights and nights, as well as their middles and ends, are different in different towns because rising and setting times are different, it is impossible to determine the same moment of time relative to the part of daylight, or the night, that has passed, because it is simultaneously different in both of them unless the rising of the sun and its setting coincide with the points of intersection of their horizons.

Then the spherical shape of the earth and the water, the

mountains and the valleys between the towns, and the diminution in the angle of vision - which ultimately is a hindrance to visual perception - prevent the arrangement in the two towns for a terrestrial signal which can be detected at the same time. So we rise from it into the air for a short distance, and say that the time of occurrence of an atmospheric incident - where its small distance from the earth may prevent its simultaneous observation in (MS 173) the two towns - can not be ascertained, because we have no previous knowledge of the occurrences of lightning, thunder, comets, or meteors. Hence we must rise from their levels to a higher level.

As to celestial phenomena, rising and setting come first, but they are not known because we are now looking for them and seeking a method for their determination. Again, the visibility of the new moon is dependent on rising and setting, so it cannot be used in this connection and, moreover, it is only known to an expert in that field. (So) then (we consider) the eclipses of the two luminaries: With regard to the sun, since its eclipse does not affect the sun itself but affects the eyes of those beholding it, and since the moon intercepting it is far away from it and is nearer to the beholders, and since, from their different localities, they make different estimates of the size of an eclipse, its duration, and the end of its visibility, the solar eclipse has not been relied upon for this investigation. The eclipse of the moon was sought, where the sun's light which falls on it is intercepted by the earth, occupying a position between them. It is known that it is a phenomenon which affects the moon itself, and that beholders

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from different localities see it truly as it is, and at its time. Hence it is truly more reliable, and the people of the profession have aimed at it (MS 174) for correcting longitudes, except Abū al-Faḍl al-Hirawī, one of the outstanding scholars in the profession of astronomy. In the tenth chapter of the first treatise of **al-Madkhal al-ṣāḥibī,** he stated, without thinking: «The determination of longitudes is arrived at by means of solar eclipses, because it has been ascertained that a solar eclipse

occurs when the moon interposes between the center of the earth and the sun, while we are situated at the center of the earth». He also discussed the hours in accordance with what we have stated.

Upon my word of faith, an eclipse would be formed, as it has been stated, if we were situated at the center of the earth, but we are not there. The intercepting body is so near to the earth that, at such a distance, the latter has an appreciable influence over the former, and, on account of it, its appearance changes. The interposition of the moon between the center of the earth and the sun, which is the cause of an eclipse, may necessitate the formation of an eclipse there, but there may be no trace of it from the surface of the earth. An eclipse of the sun may be seen on the surface of the earth without being necessarily due to an interposition of the moon between it and the center. This fact cannot be repudiated by saying that there is no connection between the true state of a thing and an empirical realization of it, because a critical inspection of the zījes points to its frequent occurrence, and this contradicts (MS 175) his statement.

Then I say: If we know beforehand of the formation of a lunar eclipse and we wish to determine the longitudinal difference between two towns, we make arrangements beforehand for someone in each town who can measure

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the times accurately by instruments, to obtain as accurately as possible the times of the beginning of the eclipse and its end, and those of the beginning of clearance and its end.

An eclipse does not become clear to a beholder until the portion formed of it, according to some authors of zījes, amounts to one digit, I mean a part of one twelfth of its size. Also, a limit has been set to its time; in time units it is 1;48°, and in hours it is 0;6,16. By this amount of time the beginning of the true eclipse precedes the apparent beginning, and the completion of the true clearance succeeds the apparent clearance. The author of this statement may have said so about both of them, but that is subject to enquiry and examination. I consider the amount of a digit in this connection to be excessive, because a small immersion can be seen, though the first contact between the shadow

and the moon is imperceptible. It is unlike the sun because the faculty of sight cannot resist its rays, which can inflict a painful injury. If one continues to look at it, one's sight becomes dazzled and dimmed, so it is preferable to look at its image in water and avoid a direct look at it, because the intensity of its rays is thereby reduced and one can look at (MS 176) its disc. Indeed, such observations of solar eclipses in my youth have weakened my sight.

But the edge of the circle of the shadow is not of pure darkness; it is this that gives the lunar eclipses their different colors. This is due to the passage of the moon through a zone of the shadow which is far away from the intercepting object, for it is characteristic of any shadow that its edges are well defined near it (the caster of the shadow), but where the shadow and the light intermingle a zone of some width is formed whose illumination is between that of a true shadow and that of pure light. This is visible in the shadow of every

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erected gnomon where a comparison is made between the contact of light and shadow near its base and away from it. Similarly, the shadow of the earth there, on account of its distance from the earth, is surrounded by an admixture of light and shadow, which gives the impression of smoke. The circle of the shadow is not pure, because if this were not the case, the contact of a small thing with it could then be perceived, as one can perceive the common boundary between an illuminated spot and a dark one, though the dimensions in this case are incomparable. But that (perception) is common to both beholders; the impression that one receives is identical with the impression that the other receives, or approximate to it.

We have not mentioned seasonal hours, known as «unequal» hours» (MS 177), in the reported times of both of them, because both observations are carried out at night, and unequal hours are measured by shadow instruments, which are (designed for use) in the sun (light) only. So those(reported)hours must be equinoctial.

There are three principal stands (reference planes) for them (eclipses): (a) Rising (on the eastern horizon), (b) Setting (western

horizon), and (c) the position midway between them where an eclipse occurs at midnight approximately, because it is diametrically opposite to the sun. An eclipse can only take place in one of the (following) positions: (a) the true rising position (the eastern horizon), (b) the true setting position (the western horizon), (c) the true meridian, and (d) a position, diverted from these three positions, that lies between them. The time of an eclipse is recorded as follows: (a) In hours elapsed after nightfall, or (b) in hours elapsed after midnight, or (c) in hours remaining up to the end of the night, or (d) in hours remaining up to midnight. This shows that there are seven possibilities for the time of an eclipse. If the time reported by one time recorder is combined with that of the other, since each time has seven possibilities, the total number of time combinations in pairs is equal to the sum of the first seven natural numbers from one to seven, and is given by seven times half of seven plus one, and that is twenty-eight. Every combination of two times can be interchanged between the two towns, so the number (of permutations) is fifty-six. In every one of these (arrangements), (MS 178) the latitudes of the two towns may both be known, or both of them may be

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unknown, or one of them may be known but the other is unknown. But if one is unknown and the other is known, an interchange can be arranged, and thus there are four possible arrangements for every combination. Hence the total number of arrangements is two hundred and fifty-six. This result was worked out by distribution, and there was no need for deriving it by induction. However, the logical distribution has led Abū Zakarīya Yaḥyā bin ᶜAdīy to declare that someone's dictum: (transliterated, a character for each Arabic letter:) 'NN ALQĀ'M ĠYR ALQĀᶜD can be arranged in sixteen thousand, three hundred and eighty-four different ways. Then it was realized that he had made a slip in multiplication, and it is said that it is eighteen thousand, four hundred and thirty-two. This was increased by Abū al-Qāsim al-Ḥasūlī, who claimed that the number of arrangements is twenty-five thousand eighty-eight.

Both results were increased by Abū Sahl ᶜĪsā bin Yaḥyā, the Christian, who stated in a letter to me that the number of arrangements is one hundred and twenty-eight thousand thousand thousand and four hundred and fifty thousand thousand and five hundred and sixty thousand (i.e. 128,450,560,000). Also, he wrote to me recently and claimed that he had obtained some extra arrangements which make the total a multiple of this number, and has promised to send me his work on that problem.

These combinations are (MS 179) distinguishable, in the sense that those related to midheaven do not require a knowledge of the latitude of both towns, or of one of them, because a meridian is one of the horizons of the right sphere, and it has no latitude. For those related

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to it (midheaven) on the one hand, and on the other related to the horizon, a knowledge of the latitude of the town of that horizon is indispensable. For those which are related to the horizon, on both sides, a knowledge of the latitudes of the two towns of those two horizons is essential. Also, some of them are symmetrical combinations if they have similar configurations in the eastern and western quarters, on both sides of midheaven.

Those which do not require one of the two latitudes are six (in number), two are singular, and four are symmetrical, with two figures for each case. Hence the number of such cases is four. The first case: when the eclipse in both towns occurs at the line of midheaven. The second case: when it occurs in both of them before the local midnight, and there is a symmetrical combination when it occurs in both of them after midnight. The third case: when it occurs in one of them in midheaven and in the other before midnight, and there is a symmetrical combination when it occurs in one of them in midheaven and in the other after it. The fourth case: when it occurs in one of them before midnight, and in the other after it.

In the first of these four cases, if the observer in each of the two towns observes the eclipse at the line of midheaven, there is no longitudinal (MS 180) difference between them, provided the eclipse occurs in both of them in the same quarter

(of the globe). However, there is necessarily a latitudinal diffe-
15 rence between them, or else the two towns must occupy the same position, but the justification of this inference is impossible because of the mountains. Also, they cannot be in two (different) quarters by being on a single meridian circle and having a longi-
18 tudinal difference of half a revolution, because if the eclipse occurs on the midnight line of one of them, it has to be at that time on the meridian line of the other. But a lunar eclipse cannot occur on the meridian at midday. This is so obvious that it does not require any illustration.

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For the second case, let ABG (Fig. 31) represent the horizon of one of the two towns, AEG its meridian circle, E the zenith, DB an arc of the equator with pole T, and let THY be an arc of the meridian circle of the other town, point H on it the zenith, and let K be the position of the eclipse. If we draw arc TKS, arc DS is the remaining time up to midnight in town E, in the first
6 figure, and arc SY is the remaining time up to it in town T, and in the second figure they are measures of the times elapsed after midnight. The difference between DS and YS is DY, which is the arc between the two meridians of the two towns E and H, (MS 181) and hence it is a measure of their longitudinal difference.

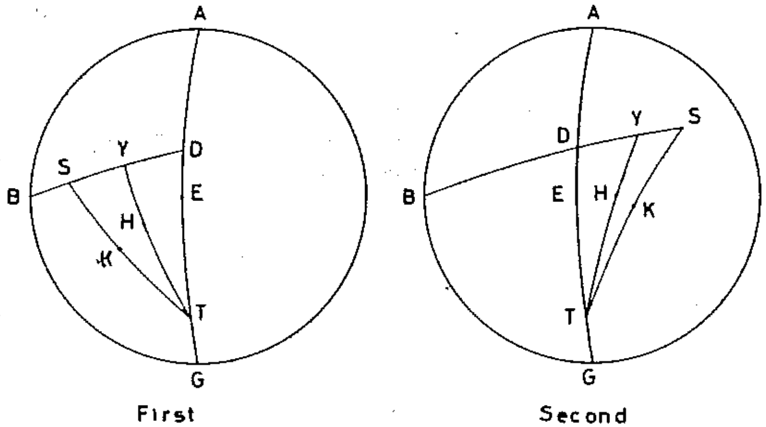

Figure 31

9 And it is known that, if the two remaining times up to the meridian are the same in both towns, or if the times elapsed after midnight are the same, then both towns are on the same meridian. Thus there is no difference in longitude between them, and the case becomes a reversion identical with the first one.
12 For the third case, let the eclipse be on the meridian of town H, then the remaining time up to midnight in town E, in the first figure (Fig. 32), and the time elapsed after midnight in Town E, in

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the second figure, is given by arc DS, and this is identical with DY which is the difference in longitude between them. (MS 182)

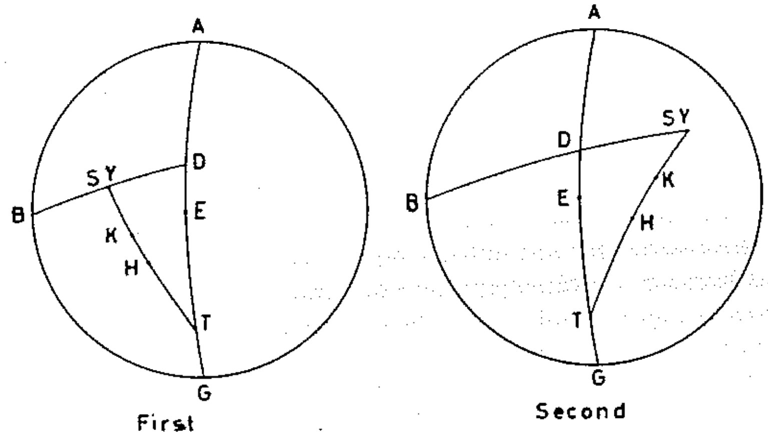

Figure 32

3 For the fourth case, let TKS (Fig. 33) fall between the meridian of E and that of H. Then YS is the time elapsed after midnight in town H, and SD is the time remaining up to midnight in town E, in the first figure; but in the second figure SY is
6 the remaining time up to midnight in town T, and SD is the time elapsed after midnight in town E, and their sum DY is the difference between the two longitudes.

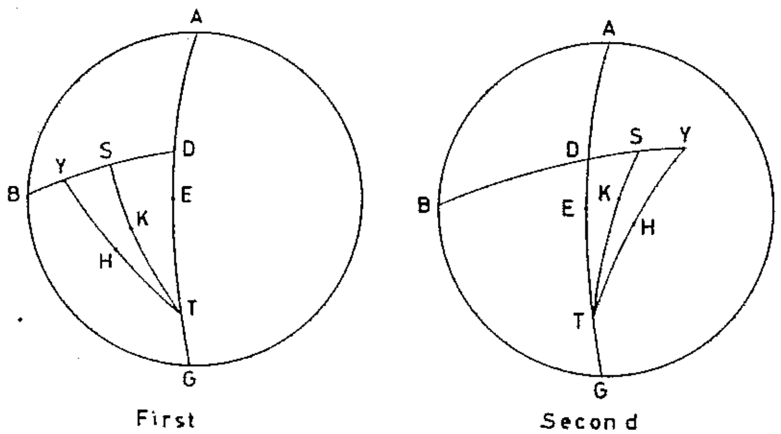

Figure 33

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These are the six distributions out of the total number of
combinations.
But those that require a knowledge of the latitude of one
3 of the two towns and not the other, are twelve (combinations),
and because of symmetry, they fall under six cases. One of the
two latitudes is required and the other is dispensed with, because
one of the two times depends upon the line of midheaven, similar
to the previous cases, and the other time is measured from the
6 horizon which has a determinate latitude. Hence the latitude is
required, (MS 183) in order to determine from it the position
and configuration.
The first of these six is when the eclipse is on the line of
midheaven in one of the two towns and the observed time in
the other is what has elpased after nightfall. There is a symmetri-
9 cal combination when the observed time is the part remaining
up to the end of the night.
The second is when the eclipse is on the line of midheaven
in one of them, and on the eastern horizon in the other. There
is a symmetrical combination when it is on the western
horizon.

12 The third is when the observed time for one of them is the
part remaining up to midnight, and in the other, it is the part
that has elapsed after nightfall. There is a symmetrical combi-
nation when the observed time in one of them is the part that has
elapsed after midnight, and in the other it is the part remaining
up to the end of the night.
15 The fourth is when the observed time in one of them is the
part remaining up to midnight, and it (the eclipse) is on the
eastern horizon in the other. There is a symmetrical combination
when the observed time in one of them is the part that has
elapsed, after midnight, and it is on the western horizon in the
other.
18 The fifth is when the observed time in one of them is the
part that has elapsed after nightfall, and in the other it is the
part that has elapsed after midnight. There is a symmetrical
combination when the observed time in one of them is the part
remaining up to midnight, and, in the other, it is the part remain-
ing up to the end of the night.

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The sixth is when it (the eclipse) is on the eastern horizon
in one of them and the observed time, in the other, is the part
that has elapsed after (Ms 184) midnight. There is a symmetrical
3 combination when it is on the western horizon in one of them,
and the observed time, in the other, is the part remaining up to
midnight. These are the six cases for the twelve distributions,
where each case includes two symmetrical combinations.
For the first of these, let K (Fig. 34) represent the eclipse
6 on the meridian of town H. We draw the parallel of declination
of the eclipse, which is KZ, and we draw TZL. Hence SL, which
is similar to KZ, is the known time after nightfall in city E, in the
first figure, and it is the known time remaining up to the end of
the night in the second figure. BL is the equation of daylight
9 of the eclipse in town E, and because we need its amount, the
latitude DE must be known to us. If we know BL, we observe
that, if the parallel of the eclipse is a northern one, and we
subtract it (BL) from the elapsed time SL, or if it is a southern
12 one and we add it (BL) to it (SL), we obtain SB whose comple-

ment SD is identical with DY, the longitudinal difference between the two towns. And it is known that if the parallel of the eclipse coincides with the equator, then the arc described is the complement of the longitudinal difference. (MS 185)

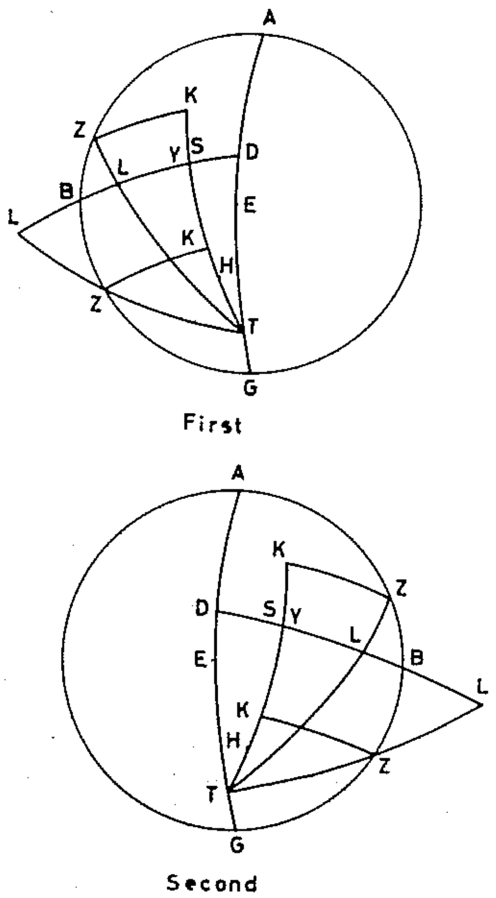

Figure 34

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For the second case, let K (Fig. 35) represent an eclipse which occurs simultaneously on the horizon of town E, and on

the meridian of town H. It is known that BL is the equation of
3 daylight of the eclipse in town E. So when the parallel of the eclipse is a northern one we add it (BL) to the quadrant DB, and when it is a southern one we subtract it (BL) from it (DB), and thus we obtain DY, the longitudinal difference between the two towns. If the parallel of the eclipse is on the celestial equator, the longitudinal difference between the two towns is a complete quadrant. (MS 186)

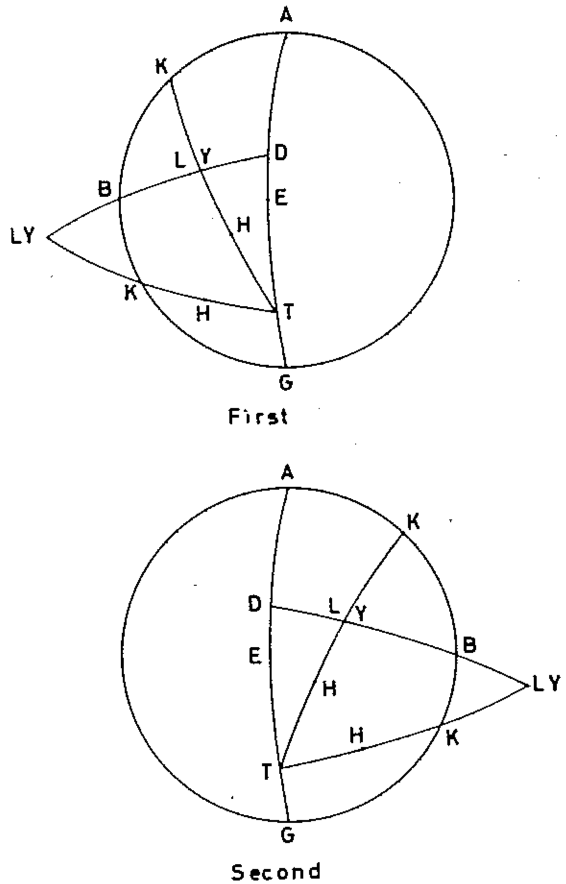

Figure 35

For the third case, let SL (Fig. 36), which is similar to KZ in the first figure, represent the known time that has elapsed after nightfall, and the known time remaining before the end of the night, in the second figure. Also, let SY be the known time remaining up to midnight in town H in the first figure, or the known time elapsed after midnight in the second figure, and let BL be the equation of daylight of the eclipse in town E. If

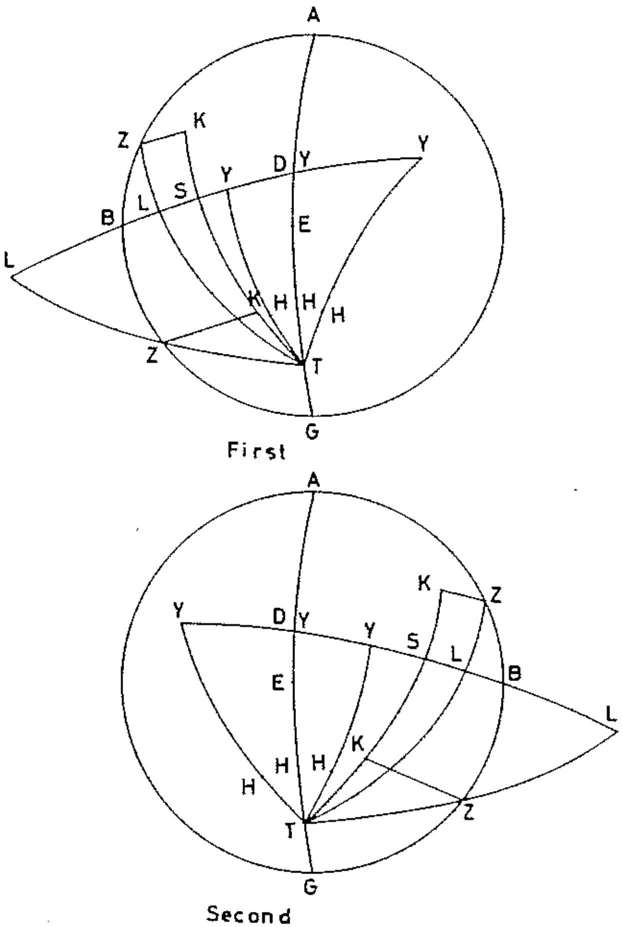

Figure 36

the parallel of the eclipse is a northern one, we subtract it (BL) from SL; and if it is a southern one, we add it (BL) to it (SL), and thus we obtain SB. If the parallel of the eclipse is on the equator, the elapsed (time) is SB itself. We add it (SB) to SY. If the sum (YB) is a complete quadrant, the two towns are on the same meridian and there is no longitudinal difference between them. But if this is not so, the difference between it (YB) and the quadrant is the longitudinal difference between the two (towns). (MS 187)

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For the fourth case, let K (Fig. 37) represent the eclipse on the horizon of town E, and let LY, which is similar to MK, be the time remaining up to midnight at town H in the first figure, and the time elapsed after midnight in the second figure. If we subtract BL, the equation of daylight of the eclipse, from LY, if the parallel is a northern one, and add it (BL) to it (LY), if the parallel is a southern one, we obtain YB, whose complement DY is the longitudinal difference between the two towns. And it is known that if the eclipse is on the equator, then YB, the time remaining up to the meridian of town H, or the time that has elapsed from it, is the complement of the required DY.

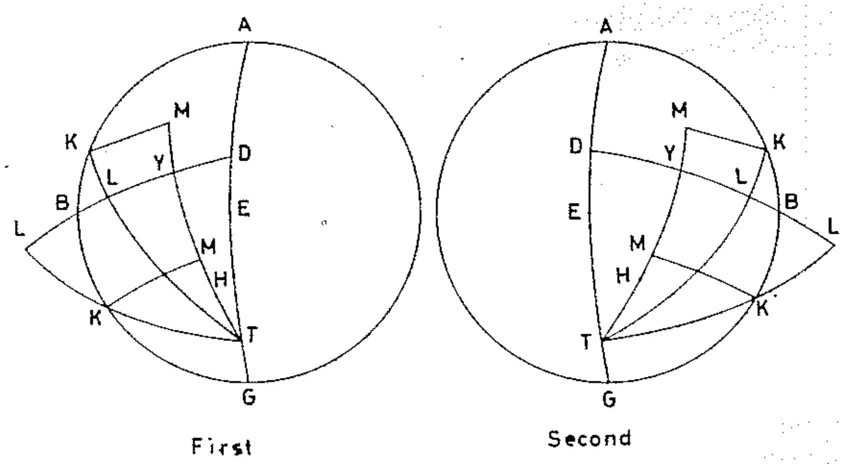

Figure 37

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For the fifth case, let SL, which is similar to KZ, be the time elapsed after nightfall in town E in the first figure, and the time remaining up to the end of the night in the second figure, and let SY be the time elapsed after midnight in town H in the first figure, and the time remaining up to midnight in the second. Let BL be the equation of daylight of the eclipse in town E. (MS 188) If the parallel is a northern one and we subtract it (BL) from SL, or if it is a southern one and we add it to SL, we obtain SB. If it is on the equator, then SB itself is known, by hypothesis, instead of SL. The difference between SB and SY is YB, which is the complement of DB (DY), the longitudinal difference between the two towns.

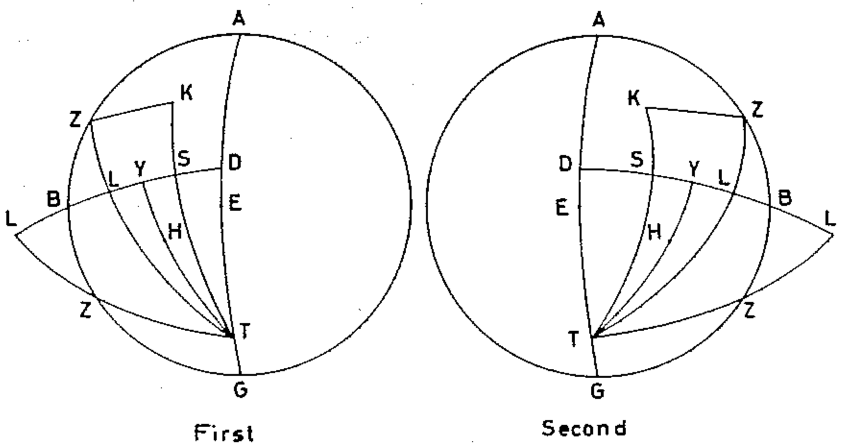

Figure 38

For the sixth case, let K (Fig. 39) represent the eclipse on the horizon of town E, and LY the time elapsed after midnight in town H in the first figure, and the time remaining up to midnight in the second figure, and let BL be the equation of daylight of the eclipse. If it (the parallel) is a northern one, we add it (BL) to LY; and if it is a southern one, we subtract it (BL) from

it (LY). If it is on the equator, the given datum LY is YB itself. If we add YB to the quadrant DB, (MS 189) we obtain DY, the longitudinal difference between the two towns.

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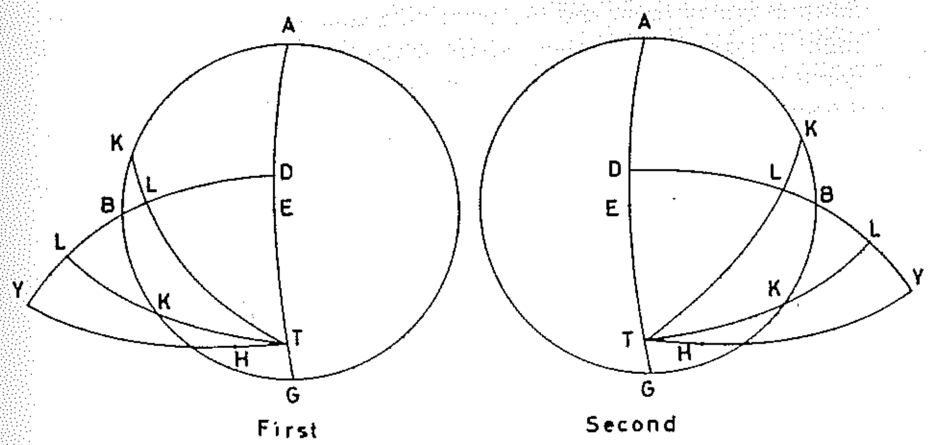

Figure 39

Those are the twelve combinations which, on account of symmetry, have been limited to six cases. Hence, from the total of twenty-eight combinations, there remain ten, eight of which can be paired by symmetry; the remaining two are simple ones. Therefore the cases of this kind are six. First, when the times recorded at both towns are those which have elapsed after nightfall, and there is a symmetrical combination when the time recorded at both of them are those remaining up to the end of the night. Second, when it occurs in both of them on the eastern horizon at nightfall, and there is a symmetrical combination when it occurs in both of them on the eastern horizon at the end of the night. Third, when it occurs at one of them on the eastern horizon and at the other after the lapse of some time from nightfall, and there is a symmetrical combination when it occurs (MS 190) in one of them on the western horizon and the time recorded in the other is

that remaining up to the end of the night. Fourth, when the recorded time at one of them is that elapsed after nightfall and it occurs in the other on the western horizon, and there is a symmetrical combination when the time recorded in one of them is that remaining up to the end of the night and it occurs in the other on the eastern horizon. Fifth, when the elapsed time after nightfall is taken at one of them and the remaining time up to the end of the night is taken at the other. Sixth, when it occurs on the eastern horizon at one of them, and on the western horizon at the other. Those are the six cases.

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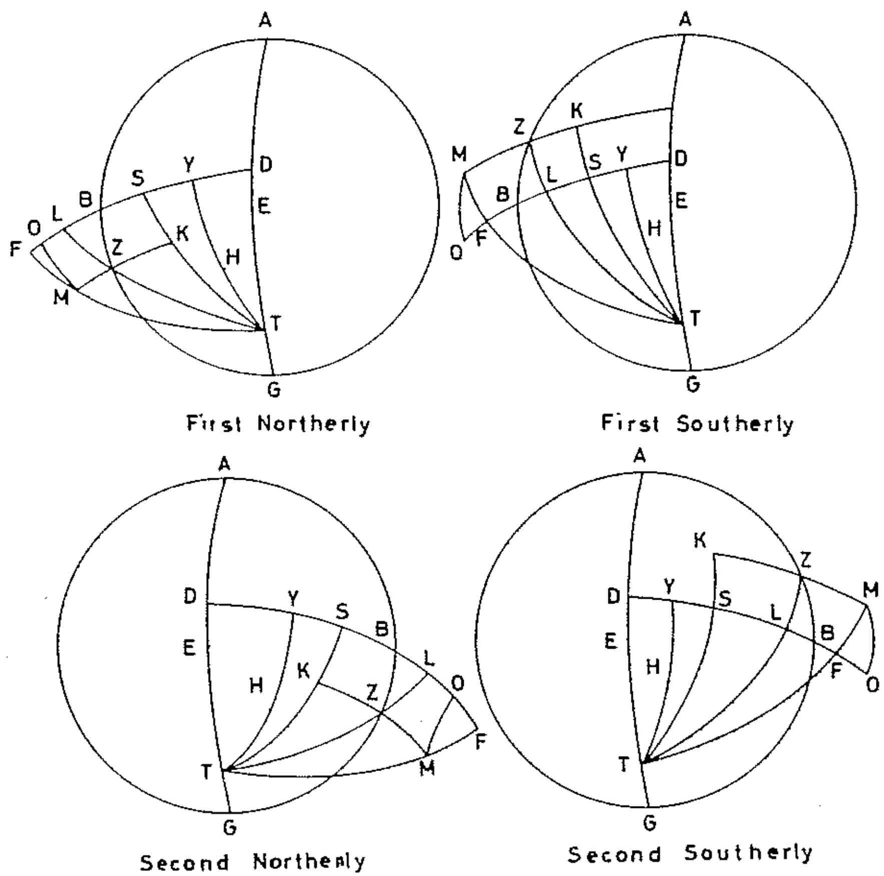

Figure 40

For the first case, let MO (Fig. 40) be part of the horizon of H, and let us draw KZM, the parallel of declination of the eclipse, and the arcs TZL and TMF. So the time elapsed from nightfall in town H is given by LF, which is similar to ZM; and that in town E is given by SF, which is similar to KM. To avoid obscurity by the use of many arcs, we give separate figures for a southern parallel. Further, it is obvious that BL is the equation of daylight for the eclipse at town E, and that OF is the equation of daylight at town H, and that they are equal if the latitudes DE and YH are equal, but that they are unequal if the latter are unequal. Since SF and SL are known, their difference LF is also known. So if BL and OF are equal, or if the eclipse were at the equator, LF would be the longitudinal difference between the two cities. Because LF is equal to OB and each of the arcs DB and YO is a quadrant, and since YB is (MS 191)

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common to both, we have, on subtracting it, that BO is equal to DY. If they are unequal and the eclipse is not on the equator, we add to LF the equation of daylight of the eclipse at that town where the time elapsed from nightfall is greater than that in the other, I mean OF, and there remains LO. Then we subtract from the result the equation of daylight at the other town, which is BL, and the sum thus obtained, which is BO, is equal to DY. (MS 192)

For the second case, we know that the eclipse occurs necessarily at an intersection of the horizons of the two towns. If the sun has zero declination, there is no longitudinal difference between the two towns, because the intersection takes place at the rising and setting points of the equinox, and if they are on the same meridian, they must necessarily have a latitudinal difference. If the eclipse has a declination, let KM (Fig. 41) be part of the horizon of town H; then BS is its equation of daylight on the horizon of town E if the declination is northerly, and SM is the equation of daylight on the horizon of town H. Their sum, BM, is equal to DY, the longitudinal difference. If the declination is southerly, then LB is its equation of daylight on the horizon of H, and the difference between them is the required BM. (MS 193)

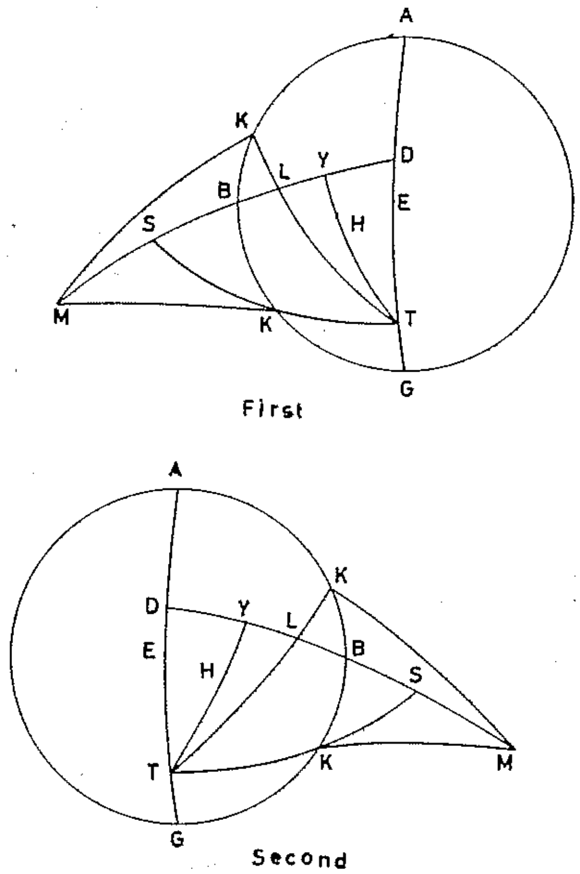

Figure 41

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For the third case, let K (Fig. 42) represent the eclipse on the horizon of E, and OM part of the horizon of H. Then LF is the time elapsed from nightfall in town H in the first figure, and the remaining time up to the end of the night in the second figure. Also, BL is the equation of daylight for the eclipse on the horizon of E, and OF is its equation of daylight on the horizon of H. The required arc is OB, because it is

equal to DY; but whenever the latitudes of the two towns are equal, BL is equal to OF; or if the eclipse is on the equator, LF would be the time elapsed or that remaining (and) equal to OB. If the latitudes are different and the eclipse has a northern declination, we add BL to LF, then we subtract OF from the sum, and if it has a southern declination, we add OF to LF. Then we subtract BL from the sum obtained, and thereby get OB, which is equal to the longitudinal difference. (MS 194)

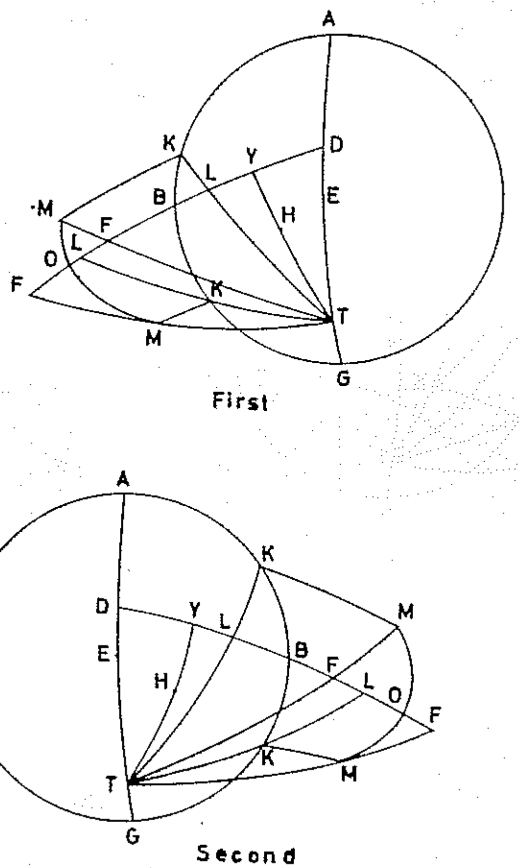

Figure 42

For the fourth case, let SL (Fig. 43), which is similar to
12 KZ, be the time elapsed from nightfall in the first figure, or the time remaining up to the end of the night in the second figure,

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and let KO be part of the western horizon of town H. The eclipse
3 K is on that horizon, and SO is its equation of daylight in town H, and LB is its equation of daylight in town E. For a northern declination we subtract BL from SL, and for a southern one we add BL to SL, and thereby we obtain BS whose complement is SD. Then we add DS to SO and thereby get DO. Its
6 sum with the quadrant OY is DY, the longitudinal difference between the two towns.

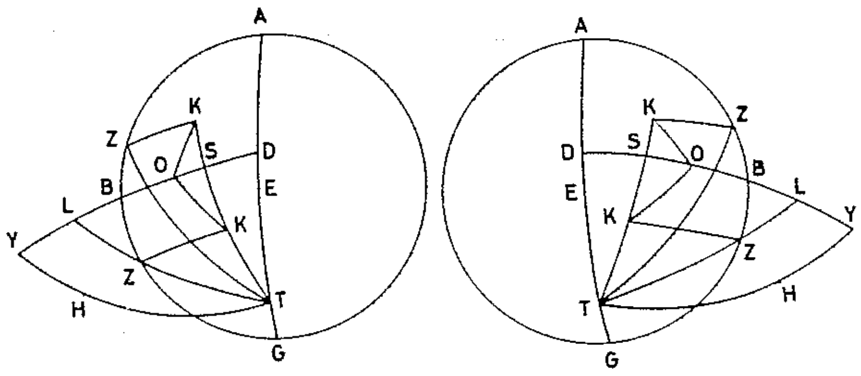

Figure 43

For the fifth case, let MO (Fig. 44) represent part of the western horizon of town H. Then SF, which is similar to KM, is the time remaining up to the end of the night in town H, and FO
9 is the equation of daylight of the eclipse there. SL, which is similar to KZ, is the time elapsed from nightfall in town E, and BL is the equation of daylight of the eclipse (MS 195) there. If the declination of the eclipse is northerly and we subtract BL

from SL, there remains BS whose complement is SD; and if we
12 subtract OF from SF there remains OS, whose complement is SY. The sum of DS and SY is what is required. If the declination is southerly and we add BL to SL, the sum BS is obtained;

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and if we add OF to SF, the sum is OS. Hence we summarize and say that we add the elapsed (time) and the remaining (time), I mean SL and SF, and we obtain LF. We add the equations of
3 daylight of the eclipse in the two towns, I mean OF and BL, then we take the difference between the two sums, which is OB, and subtract it from one hundred and eighty degrees. The remainder thereby obtained is the longitudinal difference between the two towns, because we have to subtract each of the arcs
6 OS and SB from ninety, and then add the two remainders. Whether we do that, or subtract the sum from twice ninety, there remains the sum of the two complements, which is what is required.

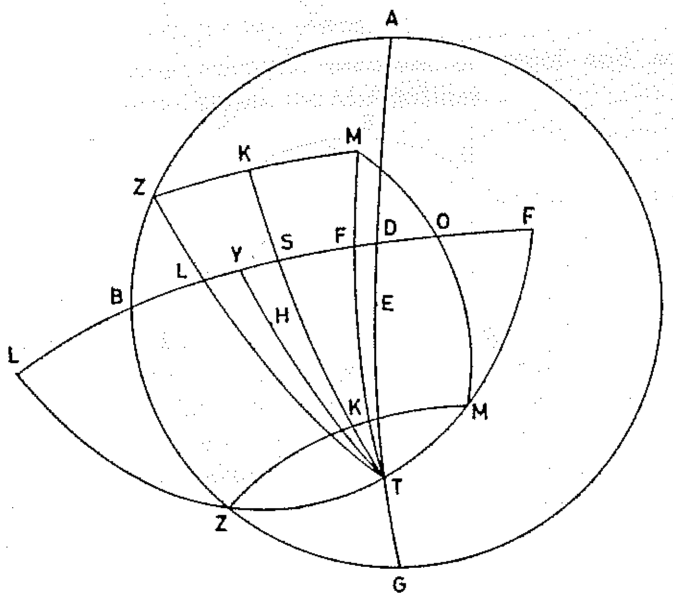

Figure 44

For the sixth case, let K (Fig. 45) represent the eclipse on the eastern horizon of (town) E, and let KF be (part) of the western horizon of (town) H. It is our intention (to separate the figure of) the northern quarter from the southern one, in order to avoid a representation of two configurations in one figure. It is known that BL is the equation of daylight for the eclipse in town E, and LF is its equation in town

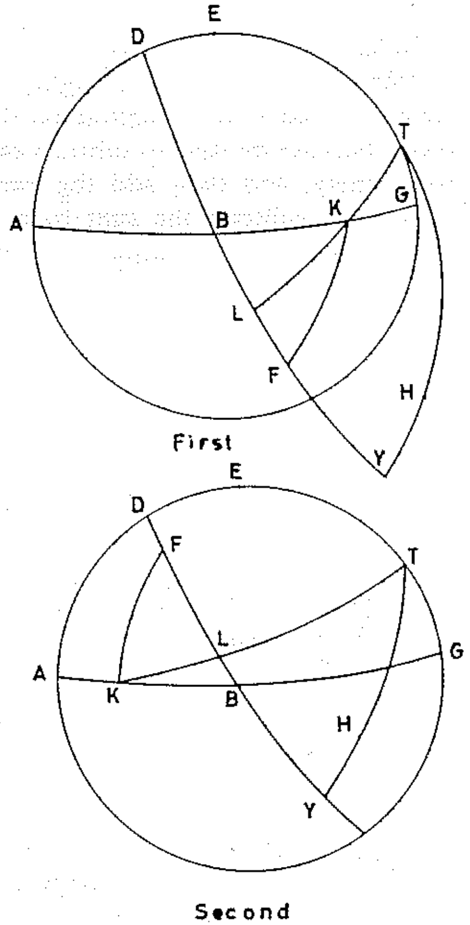

Figure 45

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H. If both (towns) are northerly and we add their sum, which is BF, to half a revolution, which is YF plus BD, we get the sum DBY, which is their easterly longitudinal difference. The part remaining of a complete revolution is their westerly longitudinal difference. If both are southerly, as in the second figure, and we subtract their sum (BF) from half a revolution, there remains their easterly longitudinal difference, because DY is the sum of the quadrant YF, and DF, which is the complement of BF. Hence DY is less than half a revolution by the magnitude of BF. It is preferable to adopt the lesser of the two distances. (MS 197)

In this case, the feet of people in the two towns are diametrically opposite. In the inhabited part of the world at the present time the Chinese and the Andalusians, in particular, are antipodal because their longitudinal difference is approximately half a revolution, but the erect human bodies (at these two localities) are not collinear, because for diametrical collinearity, there ought to be latitudinal equality as well as that longitudinal difference between the two towns.

By avoiding scientific methods of proof and adopting haphazard contradictions, the Muᶜtazilites

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have resorted to raising doubt and pure suspicion, as is evinced by their maxim; «I have not denied». Their main object in a controversy is to instil doubt in (people's) minds; it is not a critical enquiry to distinguish truth from falsehood. They would be stunned and frightened by this discourse; they can hardly imagine it, and, on being perplexed by it, they would resort to blind sophistry. They would be harrowed by suspicion whenever they hear something contrary to their point of view, before (MS 198) attempting to acquaint themselves with it or thoroughly understand it. For instance, Abū Hāshim, the leader of their school - God cure him! - has condescended to look into a book by Aristotle, entitled: «The Heaven and the World» (De Caelo). He read the part of it where the author had mentioned the circularity of water and discussed it fully in several folios, and that water

takes the shape of the containing vessel, that it takes on a square shape in square vessels, pentagonal in pentagonal ones, and circular in circular ones. What an excellent rebuff was given to him by Abū Bishr Mattā bin Yūnis al-Qinā'ī, for nothing else would have been quite as good! That was at a gathering of people where Abū Hāshim declared that he had refuted the book **De Caelo**, and then Abū Bishr took some saliva from his mouth on his middle finger and made him taste it, adding: «For God's sake, open your mind; this needs some salt!»

If I had been in his place, I would have chanted the call to prayer into his ear, and would have bitten his thumb to wake him from his stupor. It is useless to talk to them; it is, indeed, a waste of time and life. They (the Muᶜtazilites) regard their theologians - with all their error and absurdity - to be more deserving of honor than one who travelled outside Greece and devoted himself, unlike their leaders, to the search for truth.

Those are the twenty-eight combinations; I have counted them one by one. I have left out

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equatorial regions and southern latitudes, (MS 199) and discussed northern countries only, because I am relying on the intelligence of those who understand this discussion to be able to imagine, similarly, what I have left out. In actual practice, we prefer the first kind (of combination), which is related to midnight, so that we can dispense with the latitudes of the two towns, and the position of the sun and its declination. Also, we do not have to compute the equations of daylight, because the use of sines engenders errors which become appreciable if they are added to errors caused by the use of small instruments, and errors made by human observers.

Now I will summarize what I have given in detail and say: If we are given the time, recorded in each town, relative to midnight, we look (i.e., we examine the following cases). (a) If it is, at each of them, on the line of midheaven, the two towns are on the same meridian and there is no longitudinal difference between them. (b) If it is midnight at one of them but before midnight at the other, the first town is east of the second by an

amount equal to the advance of the eclipse ahead of its midnight. (c) If it is after midnight at the other, then it is east of the first by the amount of lag of the eclipse behind its midnight. (d) If it is before midnight at both of them, the longitudinal difference between them is the difference between the hours remaining up to the respective midnight at each of them, and the one whose remaining time is the bigger is west of the other. (MS 200) (e) If it is after midnight at both of them, the longitudinal difference between them is the difference between the hours elapsed after the respective midnight at each of them, and the one whose elapsed time is the bigger is east of the other. If there is no difference between the remaining times or the elapsed times, then there is no longitudinal difference between the two towns. (f) If it is after midnight at one of them and before mid-

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night at the other, the longitudinal difference between them is the sum of the time elapsed after midnight at the one and the time remaining until midnight at the other, and the town where the eclipse occurs after midnight is east of the other.

Those are the combinations which the observer should seek.

Further, if the time recorded by each of the two observers is referred to nightfall or the end of the night, it is possible to refer it to midnight, because the position of the sun is known. Then it is dealt with according to this analysis. What has been mentioned in the above enumeration of all the combinations would be too long to compute accurately. By that which I have said about the declination of the eclipse - since the declination of the moon is unreliable because of its rapid variation - I mean the declination of the point (on the celestial sphere) opposite the sun, which is known and is dependent on the middle of the eclipse. It is possible, however, to obtain (MS 201) the apparent declination of the moon at the time of the eclipse.

Some people have said that the beginning of an eclipse is imperceptible at the beginning of nighttime and that the end of clearance is imperceptible at the end of nighttime. Let the semicircle ABGD (Fig. 46) represent the part of the sun's orb

above the true horizon which is represented by AED, and the semicircle KLM the earth's hemisphere. We draw BLG tangent to the earth and parallel to AD, and hence it is in the apparent horizon. But, relative to the sun's orb, the separation between the two horizons is given by AB which is of a small imperceptible order; and its amount is given by angle AEB, which is less than three minutes.

Further, let ZHTY represent the moon's orb. Then HZ, relative to the moon's orb, is perceptible, because if it rises at Z, according to computation, it will not be perceived until it reaches H,

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and the magnitude of angle HEZ may exceed five sixths of a degree.

If we suppose that the beginning of an eclipse takes place at the beginning of nighttime, then the sun would be at D, the center of the shadow at Z, and its radius would be above the true horizon. If it happens that the sun is receding from the earth, then that will increase the width of the shadow. If it also happens that the moon is receding, so that its parallax is diminishing, and if the maximum shadow, known as the nodal orb, and the minimum (MS 202) parallax are both taken into consideration, then it is not remote that the first contact of the eclipse would be above the earth. The nearer the moon is to the earth, the greater is the width of the shadow. There is a sort of correspondence in this case and, in particular, if a recession of the sun from the earth contributes to the width of the shadow. Thus what has been claimed in this connection is a remote thing, if it ever takes place. Similarly for the end of clearance, if the sun is supposed to be at A and the center of the shadow is at Y, then the contact between the moon and the shadow would be above T.

Moreover, Ptolemy has pointed out in the fifth treatise of his book, **De Optica,** that rays of light are refracted at the surface of separation between the air and the ether. This is the cause of perceiving a thing in the east before it reaches the apparent horizon, and of perceiving it in the west after its departure from (MS 203) it.

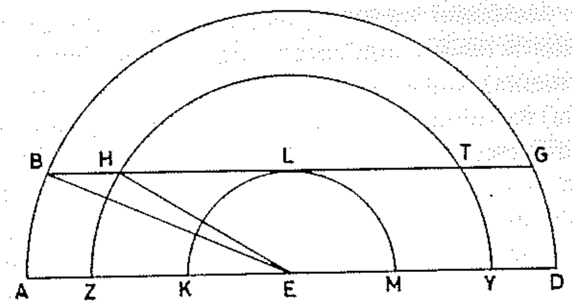

Figure 46

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It is the duty of the two observers of an eclipse to obtain all its times, so that every one of these, in one of the two towns, can be related to the corresponding time in the other. Also, from every pair of opposite times, that of the middle of the eclipse and that of the middle of its duration must be obtained. By a pair of opposite times, I mean, a pair like the time of the beginning of an eclipse and that of the end of clearance, or a pair like the time of totality of an eclipse and that of the beginning of clearance where each part of the epithet is the contrary of its corresponding part: beginning is opposite end, and eclipse is opposite clearance. They will serve for what is required, since there is a big difference between an easy theoretical investigation and a practical application of it.

The observation of these times is not made by observation of the moon and, therefore, details of its motions and positions are not required. They are the times of an event which is perceived simultaneously by peoples in distant lands, and they obtain that time by different methods.

Some of them measure it (time) with precision by continuous motions which are empirically equal in equal times and, as a rule, this has been done by the use of water. But it is subject to variation in many respects. For instance, the purity and density depend on its sources, nevertheless they are attributed to water itself because they are permanent properties of it. Also, it is

subject to accidental variation, by variation in the quality of the air. As water is readily influenced by air because of the adjacency (of the elements), and as its pressure on air increases with increase in (MS 204) its volume and decreases with its decrease, and so on, man has preferred the motions of sand to it.

Some of them measure it (time) with precision by observation of the altitudes of the stars and their azimuths, and all that depends on an accurate determination of the position longitudinally opposite to that of the sun. If he (an observer) observes it by water or by sand,

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it is done by measures of fixed volumes or by balances, which are well known, and there is no need for words about them. However, if altitudes of fixed stars are observed, though they are numerous, the corroboration of evidence obtained from some of them with that obtained from others leads to better accuracy. Either its altitude only is observed, or the azimuth of its altitude is observed, or both of them are observed simultaneously. If there had been no confusion in the zījes about that, I would not have discussed it here, but a practical worker may not be competent enough to distinguish a sound exposition of that matter from a disorderly one.

When he (an observer) observes the altitude of a star, he multiplies its sine by the star's day-sine, and he divides the product by the sine of the star's meridian altitude. If he subtracts the quotient from the day-sine, there remains the versed sine (of the time) between the star's observation and that of its transit. If he takes the arc of this versed sine and subtracts it from the ascension of its (ecliptic) degree of culmination in the right sphere when the altitude is easterly, but adds it when it is westerly, he obtains the ascension of the (ecliptic) degree of upper midheaven in the right (MS 205) sphere at the time of observation.

To prove that, let the circle ABG (Fig. 47) represent the horizon, AEG the meridian line, EB the east-west line, and DZ the common line between the plane of the horizon and that of the parallel of declination. Let THZ be the triangle of daylight. Then TH is the sine of its meridian altitude, and HZ is its day-sine. Let OLF be the triangle of time. Then LO is the sine of the altitude

at the time, and since the two triangles are similar, the ratio of OL to LF is as the ratio of TH to HZ. Hence, if we multiply the first

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by the fourth and then divide the product by the third, the quotient obtained is the second, which is LF. We draw LK parallel to FZ. Then KZ equals LF and (on subtraction from HZ) there remains HK, the versed sine of the arc of the day-circle (passing through H and L). This arc is a measure of the time remaining for the star to reach the meridian, if the triangle of time is east of the triangle of daylight; but, if it is west of it, it is a measure

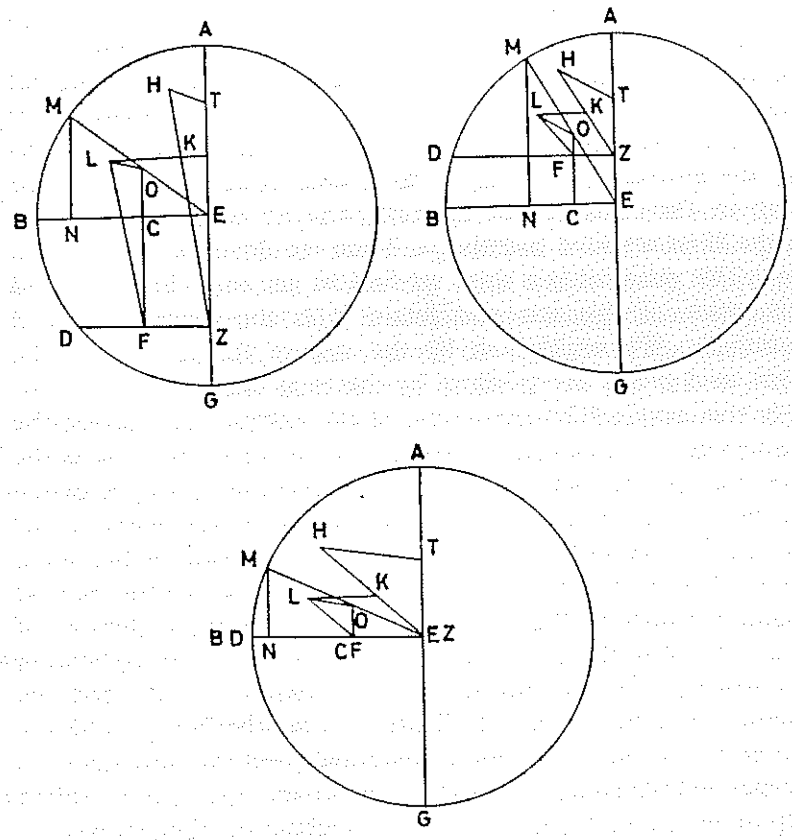

Figure 47

of the time that has elapsed after transit. The circle through the pole of the equator and the star L intersects the ecliptic at the star's degree of culmination, and it intersects the equator at the star's right ascension. Thus an arc (on the equator) similar to arc HL is obtained between the star's meridian and the local meridian. The ascension of the (ecliptic) degree of midheaven leads by it, (MS 266) if the star has not reached the meridian yet. Therefore, if we subtract the arc from the ascension of the (ecliptic) degree of

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culmination, we reach the point of intersection of the equator with the meridian. Also, the ascension of upper midheaven lags behind by its amount if the star has already passed the meridian. Hence, if we add that arc to the ascension of the (ecliptic) degree of culmination, we reach the aforesaid point. (MS 207)

If the observed coordinate is the azimuth of the star, without the altitude, we multiply the cosine of the latitude of the town by the cosine of the azimuth, and we retain this product as the «first retained (number)». Then we divide it by the total sine. The quotient is a sine; we find its arc sine. Then we find the cosine of this arc, and retain it as the «second retained (number)». We multiply it by the sine of the latitude of the town, and divide the product by the total sine. Then we multiply this quotient by the cosine of the azimuth and divide the product by the total sine. The quotient obtained thereby is the sine of an arc. We find the arc sine and retain it. Then we divide the first retained (number) by the cosine of the star's declination, and we multiply the quotient by the sine of the star's declination, and then we divide the product by the second retained (number). The quotient obtained thereby is the sine of an arc. We find the arc of the inverse sine, and if the declination is northerly, we take the difference between this arc and the retained arc, but if the declination is southerly, we add the two arcs. The result of either operation gives the time between the star and (its passage across) the meridian, either pre-meridian or post-meridian. If the star has zero declination, then the arc retained for it is the time remaining until its transit or the time that

has elapsed after it.

To prove that, let ABGD (Fig. 48) represent the meridian circle, AED the horizon with (MS 208) pole at S, EBL the celestial equator with pole at G. Let point K represent the star. From S we draw the altitude circle through K, and let this be represented by SHZ. Then EZ is the displacement of azimuth from the east-west line. With H, the point of intersection of the altitude circle with the equator, as pole, and with distance equal to the side of the square, we draw the quadrant DML (GML), and we extend to it HBL and HSM.

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In this sector, the ratio of the sine of SG, the colatitude of the city, to the sine of GM is as the ratio of the sine of angle GMS, the right angle, to the sine of angle MSG, which has the magnitude of the complement of the azimuth, which is ZA. So arc GM is known. Since we need the product of the sine of GM times the total sine, and it is equal to the product of the sine of SG times the sine of angle MSG, we have retained the latter product as our first retained (number), so that it can take the place of the former product when the time comes for it. Further, the ratio of the sine of SH, it is called the complement of the intermediate altitude, to the sine of the latitude of the city, is as the ratio of the sine of HM, the quadrant, to the sine of ML, the complement of GM. Hence, the sine of SH is known. Similarly, we have retained the sine of ML as the second retained number, because we shall need it later on. Further, the ratio of the sine of SH to the sine of HB is as the sine of SZ, the quadrant, to the sine of ZA, the complement of the azimuth. Therefore arc HB, which is retained for its importance in this discussion, is known. Also, the ratio of the sine of KG, the complement of the star's declination, to the sine of GM, is as the ratio of the sine of angle GMK, the right angle, to the sine of angle GKM. So the product of the sine of GM times the total sine is the first retained (number). So the sine of angle GKM is known, and its ratio to the sine of ML, the second retained (number), is as the sine of TH to the sine of KT, the star's declination. So TH is known. In the first and second figures, the difference between

TH and HB is arc TB, which is the time remaining for the star to reach the meridian or the time that has elapsed after its transit. In the third figure, it is their sum which is equal to arc TB, but in the fourth figure arc HB is arc TB itself. The derivation of the ascension of upper midheaven from this arc is similar to that given above in the article on the altitude. (MS 210)

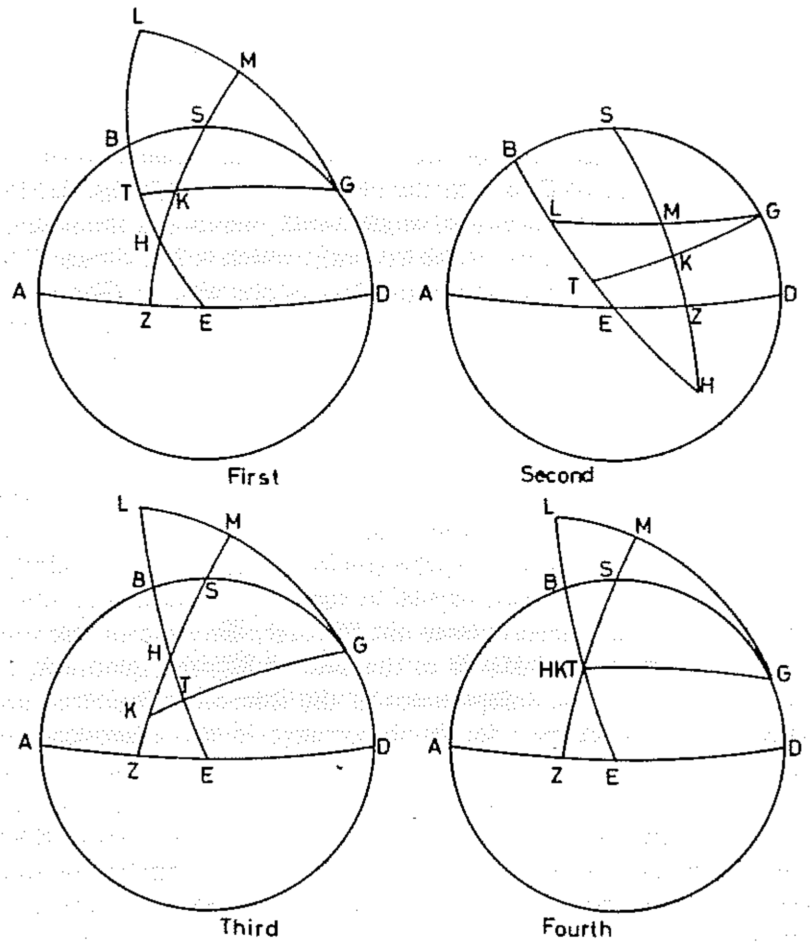

Figure 48

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If both the altitude and the azimuth are observed, we draw in the figures (for the case) of the altitude the line EOM from the center to O, the foot of the vertical perpendicular from the star, and we drop the perpendicular MN to EB. Then the ratio of EO, the cosine of the star's altitude to OC, the «share» of the azimuth, is as the ratio of EM, the radius, to MN, the sine of the azimuth. Since EO is the hypotenuse of a right triangle with legs OC and CE, if we subtract the square of the share of the azimuth from the square of the cosine of the altitude, there remains the square of EC. But EC equals KL, and KL is the sine of the (hour angle) remaining until the star reaches the meridian, or the angle that has elapsed after its transit, where the angle is evaluated

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in the parallel, I mean, where the radius of the parallel is the cosine of its declination. (MS 211) Since whatever results for us is in units of half the diameter, EO, MN, and OC will be in that scale, so it will be necessary to transform it. The ratio of KL to half the diameter of the parallel, taken as a measure of the cosine of its declination, is as the ratio of KL to half the diameter of the parallel taken as a measure of the total sine. Hence we multiply KL, which we have evaluated, by the total sine and divide the product by the cosine of the declination of the parallel. Thereby it is transformed to a sine in the parallel, and then we find the arc of the sine. From this arc we derive the right ascension of upper midheaven at the time (of observation). We take the difference between the ascension of the degree of upper midheaven at sunset and that ascension, and we multiply it by the **buht** of the sun, which is a measure of its daily variable velocity at that time. Then we divide the product by three hundred and sixty, and we add the quotient to the opposite degree of the sun at sunset. We obtain thereby its opposite degree at the time of observation, and it is this opposite degree whose declination we use in the operations mentioned above. I have obtained (books containing such operations) in which the decli-

nation of a star and its (ecliptic) degree of culmination are used. There is such a lamentable corruption in them and in the zījes that it excites (the shedding of) tears. It is not safe to take them as they are reported, (simply) because of good faith in their authors and their high standing in science. There is no inconvenience if I eradicate the cause of that confusion.

As to the declination of a star, al-Khwārizmī in his zīj, and all the adherents of (MS 212) the Sindhind, define it as the star's distance from the equator. In Ḥabash's zīj, it is defined as the declination of the star's course, and in the zījes of al-Battānī and al-Nairīzī it is defined as the star's distance from the celestial equator. We regard (here) the distance of the (ecliptic) degree of the star from the first point of Aries as a measure of right ascension. We enter it in the table (of ascensions), and we take the adjacent (number of) equatorial degrees and call it «the longitude» (**al-ṭūl**). We take the «declination» of the longitude, and note the side on which it falls. If it falls on

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the same side as the star's latitude, we add the two, but if they fall on opposite sides we subtract the smaller from the greater and there will be a remainder on the side of the greater. Then we take the lesser of the two distances of the star from the nearer solstice to it, and we take the «declination» of that distance and multiply its cosine by the sine of that remainder or the sine of that sum. Then we divide the product by the total sine. The quotient obtained thereby is the sine of the star's declination, which is either on the side of the remainder or on the side of the sum.

To prove that, let ABGD (Fig. 49) be the circle through the four poles, arc EA part of the celestial equator whose pole is G, and arc BE part of the ecliptic, whose pole is D. Then B is the solstitial point. Further, we suppose that point K represents the star, and we draw the arc DKZH. Then point Z is its degree (of longitude). Also, we draw the arc GKT. Then KT is its distance from the celestial equator, and EZ is the distance of the star's degree from the equinox. (MS 213) Since ZH is perpendicular to BE, we may regard ZE with respect to EH as a measure

of right ascension; and if we regard it as such, its corresponding equatorial degrees are represented by EH, which is the «longitude». Its declination HZ is north of the equator, and the star's latitude KZ is north of the ecliptic in the first figure, but south of it in the third one. Since HZ and ZK are arcs of the same circle, their sum in the first figure, or their difference in the second one, is equal to KH.

With H as pole and distance equal to the side of the square, we draw the (great) circle GMSO.Then its (angle ZHE) magnitude is MS and its complement is GM. Since each of the arcs OZ and OM is perpendicular to the circle ZM, point O is a pole of arc ZM. Hence MO is a quadrant, and GS is a quadrant, and on subtracting the common arc MS there remains GM equal to SO. Also, ZO is

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a quadrant (and EB is a quadrant). Hence EO is equal to ZB, the least distance of Z, the star's degree, from the solstice (B). The «declination» of ZB is SO, and the complement of this «declination» is MS, the magnitude of angle ZHE. Further, the ratio of the sine of HK, the sum or the remainder, to the sine of KT, the declination of the star with respect to the celestial equator, is as the ratio of the sine of the quadrant HM to the sine of MS. Therefore KT is known.

Again, if we like, we can draw the arc EKL. Then the ratio of the sine of DK to the sine of KL is as the ratio of the sine of DZ, (MS 214) the quadrant, to the sine of ZB. So if we multiply the cosine of the star's latitude by the sine of the lesser of its two distances from the nearer solstice, and we divide the product by the total sine, the quotient obtained thereby is the sine of KL. Hence the complement, KE, is known and its sine is the «part». Further, the ratio of the sine of KE to the sine of KZ is as the ratio of the sine of LE to the sine of LB. So if we multiply the sine of the latitude by the total sine, and we divide the product by the cosine of the arc sine, where the arc can be evaluated from our first quotient for its sine, the quotient obtained is the sine of LB. We find the arc of this sine and retain it. If the latitude of the star and the «declination» of its degree are on the same side, we add the

retained arc to the maximum declination; but if they are on opposite sides, we take the difference between the retained arc and the maximum declination, and we obtain thereby arc LA. If the retained arc is equal to the maximum declination, then the star has no declination with respect to the equator. Further, the ratio of the sine of LA to the sine of EL is equal to the ratio of the sine of KT to the sine of EK. So if we multiply the sine of the result by the «part», and we divide the product by the total sine, the quotient obtained is the sine of KT, where KT is the declination of the star relative to the equator, and that is what is required. (MS 215)

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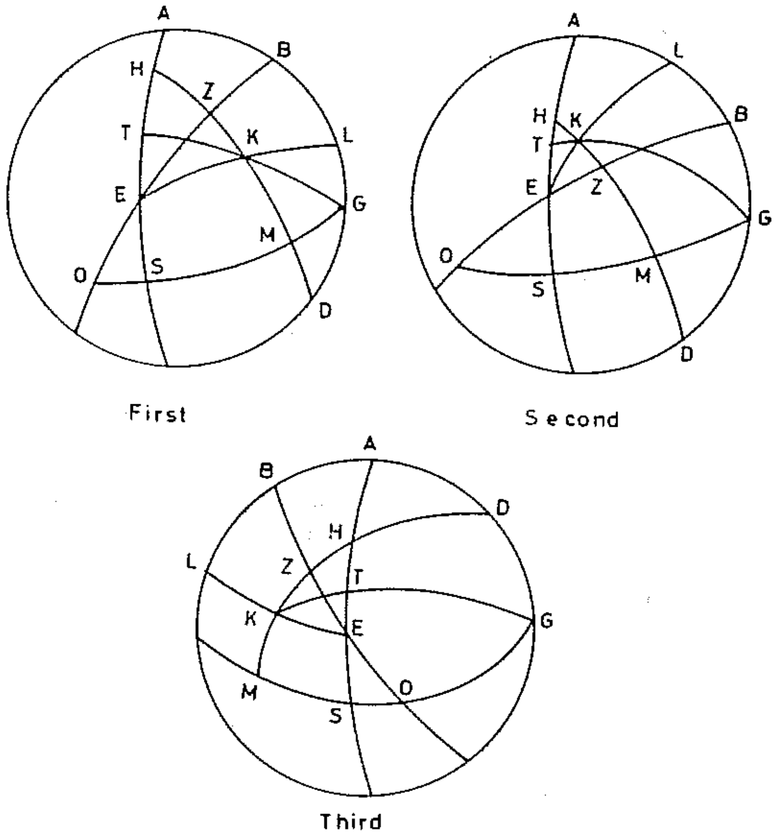

Figure 49

As to the (ecliptic) degree of the star's transit on the line of midheaven, we construct for it each of the quadrants KY and KL (Fig. 50). Then with K as pole and with distance equal to the side of the square, we draw the quadrant LYF. The ratio of the sine of FH, the complement of the equation, to the sine of HY, the complement of HK, is as the ratio of the sine of FT, the quadrant, to the sine of LT, the complement of TK. So if we multiply the cosine of the sum or the cosine of the remainder by the total sine, and we divide the product by the cosine of the star's declination (MS 216) with respect to the equator, the quotient is a sine. We find its arc sine, and we subtract it from ninety. The remainder obtained is the equation. Again, the ratio of the sine of KH to the sine of KT is as the ratio of the sine of KG to the sine of GM, which we said is equal to the «declination» of arc BZ, the lesser of the star's two distances from the solstice. So if we multiply the sine of the sum, or the sine of the remainder, by the sine of the «declination» of the lesser of the two distances of the star's degree from the solstice, and we divide the product by the cosine of the star's declination

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relative to the equator, the sine of the equation is obtained. If the degree (of longitude) of the star is in the half that goes from the winter to the summer solstice, where the vernal equinox is in the middle, and if the declination of the star is northerly, as in the first figure, or if it is in the other half and the declination is southerly, as in the third figure, we add HT, the arc of the equation, from the extremity H of the «longitude» and it will terminate at T. But if it is in the half that goes from the summer to the winter solstice, where the autumnal equinox is in the middle and the declination is northerly, as in the fourth figure, or if it is in the other half and its declination is southerly, as in the second figure, we subtract HT, the equation, from the extremity, H, of the «longitude», and it will terminate at T, where T is the extremity of the right ascension of the (ecliptic) degree of transit. If we find its arc, (MS 217) the number of equatorial degrees that we take from the adjacent entry gives us the degree of O which culminates simultaneously with it (the star).

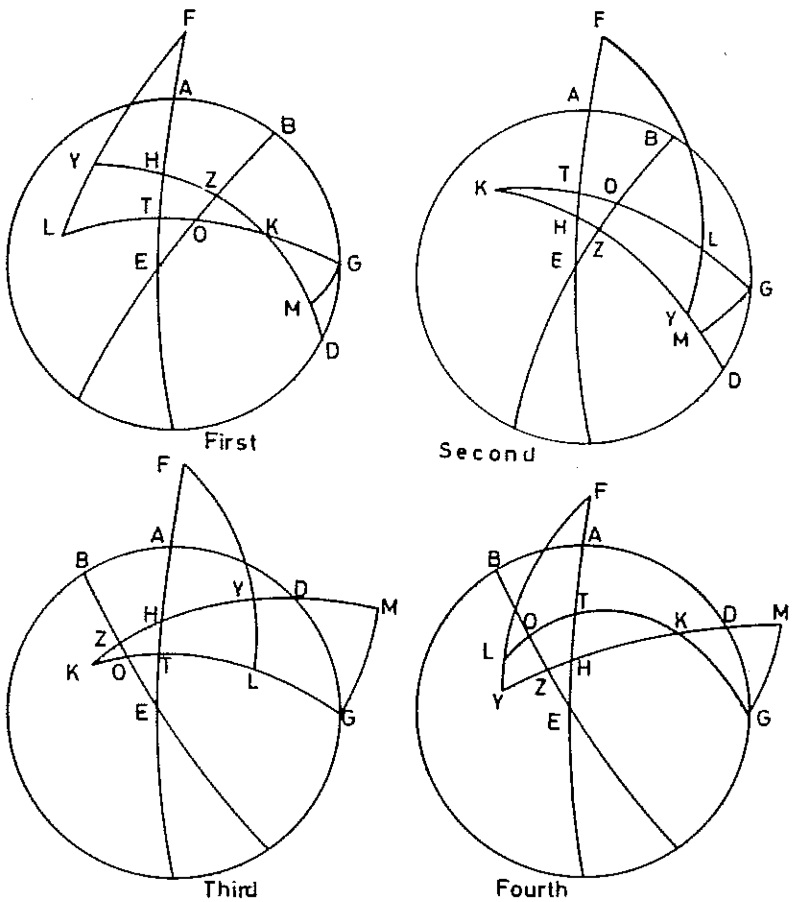

Figure 50

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I have seen a treatise on the correction of the longitude of Jurjān, which was presented to Zarrayn Kīs, daughter of Shams al-Maᶜālī, by Abū ᶜAlī al-Ḥusain bin ᶜAbdallah bin Sīnā (Avicenna). He stated in it that, when she commissioned him to do it, he had to resort to a determination of the meridian altitude of the moon, because no previous arrangement had been made

with people living in towns of known longitude, nor was there any lunar eclipse expected during that year which could be simultaneously observed by two preassigned observers. He said that he had observed it and found it to be 80;6°, but he did not give the date of his observation. Then he calculated the position of the moon at the time of its meridian observation, on the supposition that the longitudinal difference between Baghdad and Jurjān is eight degrees. He derived its latitude and its declination; and this made it necessary, according to the latitude of Jurjān which he had observed, that its altitude at that time should have been 80;4°, if it had been in that calculated «part». He deduced that the moon had passed the meridian of Jurjān; and he interpolated until he knew the «part» where, had it been there, its altitude in the latitude of Jurjān would be identical with that which was found by observation. But that was not possible unless he increased those eight degrees by one degree and a third. This makes the longitudinal difference between Baghdad and Jurjān 9;20°. Then he mentioned that he had checked that by examining the position of the moon over Baghdad at the time of observation, and that he had also observed the altitude of the moon as it touched (MS 219) the shoulder of Pegasus and other fixed stars.

This method is valid in principle, but it is difficult to apply in practice, because it is based on the authority of the zīj from which the position of the moon and its conditions were calculated. The traditional method

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for (determining) the longitude of Jurjān is more feasible. Also, the data for the moon can rarely be obtained with precision, because of the rapidity of its movement and the parallax associated with it. On account of these, hardly anything desired can be obtained (with precision). The determination of the time of the moon's transit over a town of known longitude and latitude is a long and tedious matter, and the determination of an unknown longitude would be even more so. At any rate, it is one of the scholarly methods for deducing results by what is available or possible at the time (of investigation); but though Abū ᶜAlī is renowned for his intelligence and sound intuition, he is unreliable

in a matter which requires practical experience and, in particular, in a commission ordered by Her Highness.

As to the author of some zīj, he may allege the validity of his zīj by correcting it, and regard this as a subtitute for astronomical observation. Thus he would give an instruction to observe the times of an eclipse at the town whose longitude is required, and to compute them at the town on which the zīj is based. For instance, in the zīj of Ḥabash al-Ḥāsib, he gives an instruction to compute the times of an eclipse at Baghdad, on which his zīj is based, and then to observe those times at the town whose longitude is required, and to determine the time interval between each pair of (MS 220) corresponding times. If the observed time is identical with that obtained by computation, the two longitudes are identical. Otherwise, we add the observed and the computed times and we multiply the sum by fifteen. If the observed time is earlier than that computed, we add the product to the longitude of Baghdad; but if it is later than the computed one, we subtract that product from the longitude of Baghdad, and thereby we obtain the longitude of that town. This treatise in the copy of that zīj in my possession is so corrupt that no more guidance could be gleaned from it, except what was (just) mentioned. As to the halving of the interval between the two times, it is a rule of procedure which has been adopted by calculators for the purpose of minimizing the errors of observation, so that the time calculated will be between the upper and the lower bounds. As to the addition of

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the longitudinal difference to the longitude of Baghdad when the observed is earlier than that computed, it is a rule whose sense is correct, but its phraseology leads into error anyone who does not know that sense. For if the town of observation is east of Baghdad, the addition of the longitudinal difference to the longitude of Baghdad is necessary when that town is before Baghdad and the eclipse reaches it earlier, but its hours are more than those at Baghdad. Because, although it is the same instant of time, sunset at that town is earlier than sunset at Baghdad. To an intelligently critical worker there is nothing

ambiguous in that, (MS 221) but if he is only a practical user of zījes he would think that the eclipse takes place earlier, at the town where its hours are the lesser, than at the town where its hours are the greater. A similar case, where the hours are measured from the beginning of the night, was discussed above.

Abū ᶜAlī Muḥammad bin ᶜAbd al-ᶜAzīz al-Hāshimī has narrated that a lunar eclipse occurred on Friday night, the fourteenth of Dhū al-Qaᶜda, year three hundred twenty of the Hijra, and that he computed its time at Baghdad, and then observed it at Raqqa. He found the difference between the hours to be 0;28, which is equal to 7;5 units of time, and that this is a measure of the longitudinal difference between Baghdad and Raqqa. However, there were reasons for suppressing the report of the work he cited, because the hours at Raqqa were more than those at Baghdad, and since it is known that Raqqa is west of Baghdad, the hours at the westerly town should have been less. It is possible to attribute that to the corrupt manuscript, due to the carelessness of the scribes and, in particular, to the use of the letters of the alphabet as numerals. Moreover, the latitude of Raqqa, as determined by al-Battānī, is 36;1°, and the latitude of Baghdad is 33;25°; and since the hours in both of them were measured from the beginning of the night, this case reverts

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to the first of the cases of the third distribution of combinations discussed above. Since Baghdad and Raqqa are not on the same parallel of latitude, (MS 222) the unconditional difference in the hours is not simply what is required. There must be a consideration of what was mentioned in that case concerning the direction of the declination of the eclipse, and the evaluation of the equation of daylight at both towns.

I have found in some books that the scientists measured the longitudes of towns with reference to Alexandria in Egypt by observation of eclipses. They found the hours of a certain eclipse at it to be 4;30, and those at Raqqa to be 5;20. So they subtracted the smaller from the greater, and there remained 0;50, which is their longitudinal difference.

I am not sure whether this is a genuine report of what was obtained by observation, or whether it is just an example made up for illustration, after the longitudinal difference had been obtained. Actually, it is a problem which should also be treated as in the first case of the third kind, because the latitude of Alexandria is 30;58°, and that of Raqqa is what we had mentioned.

Muḥammad bin Isḥāq al-Sarakhsī mentioned in his zīj a rule which is related to that question. He said: «Compute the times of a lunar eclipse at the Cupola, then measure them by observations in your town, and derive the equation of daylight for the degree of the moon. If the moon's half arc of daylight is greater than ninety, add the equation of daylight to the hours observed, but if it is

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less, subtract the equation of daylight from the hours observed. Then take the difference between it (the result of the addition, or subtraction) (MS 223) and that (result) computed for the Cupola. If the hours of the Cupola are the greater, add that difference to ninety, but if the hours of the Cupola are the smaller, subtract it from ninety. There remains the longitude of the town from (a zero meridian in) the east.» If the condition imposed for the addition, or subtraction, of the equation of daylight of the eclipse is reversed, that is, if addition is imposed when the half arc of daylight is less than ninety, and subtraction is imposed when it (the semi-arc of daylight) is more (than ninety), then that rule is valid, otherwise it is false.

To clarify that rule, let us repeat some of the previous configurations. Let ABT (Fig. 51) be the horizon of the Cupola, which is a locality of zero latitude. His zīj, which he regards as a substitute for observation, is based on it. Then point T, representing the north pole, is necessarily on that horizon, and the azimuth E is on the celestial equator at D. Let H represent the town of observation, and THY its meridian. Then the hours of the eclipse, computed for the Cupola, are given by the arc BS, which is similar to KZ, and the hours found by observation at town H are given by arc SF, which is similar to KM. His in-

tention is to find BO, which is equal to DY. But it is known that the equation of daylight for a northerly (declination) should be deducted from SF, and that it should be added to a southerly (declination), in order to obtain OS. The difference between it and BS is the required arc OB. Again, the half arc of daylight does not exceed ninety unless the declination is northerly, and it does not fall below ninety (MS 224) unless the declination is southerly. Therefore the equation of daylight should be deducted from the northerly, but it should be added to the southerly. A simple matter like this would not have trapped a man like Muḥammad

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bin Isḥāq if he had not been careless, but scientists often make slips in their conditional statements concerning opposite things.

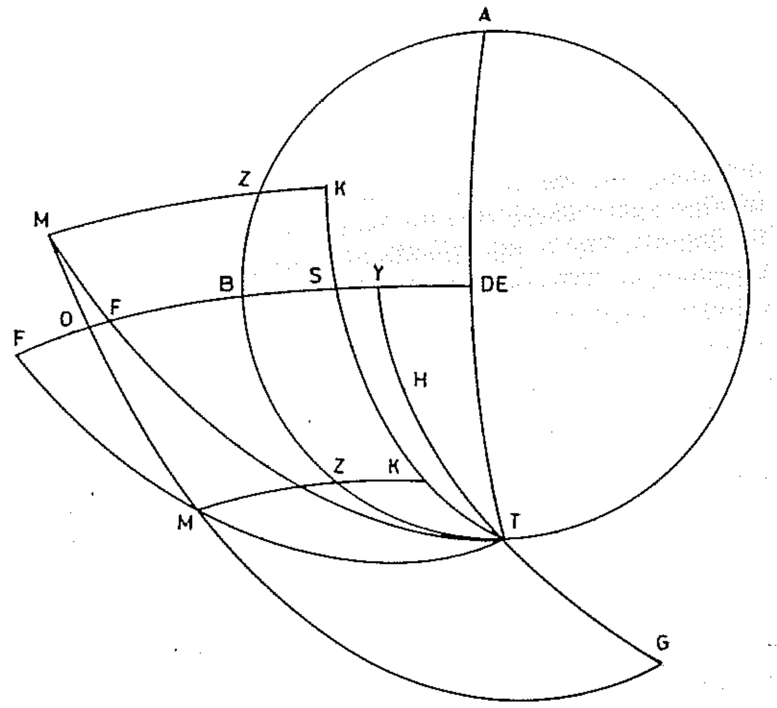

Figure 51

3 As to the Cupola, it is the middle of the inhabited world,
and its position varies according to the position of its two
extremities, as we have mentioned heretofore. We have to rely
on the reports of the easterners about it, for others do not refer
to it. They have also claimed that it is east of Baghdad by one
6 hour and a third. When he used the Cupola, he took the longitude
of the town from (a zero meridian in) the east, but there is no
harm in that, because an agreement can be reached and there
should be no discord over this matter. (MS 225)

When both the longitudes and the latitudes of two towns
9 are known, other relations connected with them become known.
They are: the distance between them, the direction of the one
relative

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to the other, and the points of intersection of the two horizons,
because this relation is essential in the study of great circles, and
horizons are such circles. They are very useful relations, on this
earth and in the hereafter.

3 Let ABG represent the horizon of town E (Fig. 52), AEG its
meridian, BD the celestial equator, and THY the meridian of
another town, where H is the zenith on that meridian. So HY is
6 its latitude, DE is the latitude of town E, and arc YD is their
longitudinal difference. Further, we draw the altitude circle
through the zenith of H. So the azimuth of H is at the intersection
of this circle with the horizon of E; arc BL is the distance of this
9 azimuth from the east-west line; arc AL is its distance from the
meridian, and arc HE is the distance between the two towns. To
determine that distance, we draw the circle DHK. Then the ratio
of the sine of HT to the sine of HK is as the ratio of the sine of
TY, a quadrant, to the sine of YD. Hence, if we multiply the
12 cosine of the latitude of the town whose azimuth is required by
the sine of the longitudinal difference, and we divide the product
by the total sine, the quotient is the sine of HK. Its arc is called
the «equated longitude». Further, the ratio of the sine of BH to the
15 sine of HY is as the ratio of the sine of BK, a quadrant, to the sine
of KD. (MS 226) Hence, if we multiply the sine of the latitude of
the town whose azimuth is required by the total sine, and we

divide the product by the cosine of the equated longitude, the
quotient is the sine of KD. The arc KD is called the «equated
latitude»; and, by means of this arc, the position of the azimuth
18 relative to the east-west line would be known. For if it is less than
the latitude of the town, the azimuth at it is south of the east-west
line; if it is more, it is north of it, and if it is equal to it, then the
azimuth is on the east-west line itself. If that is so, - I mean if it
21 is on the east-west line - then the two horizons intersect at the
south and north points of the town under consideration,

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and the equated longitude itself is the distance required. But if
that is not the case, the difference between the latitude of the
town and the equated latitude is EK, and the ratio of the sine of
3 BH to the sine of HL is as the ratio of the sine of BK to the sine of
KA. Hence, if we multiply the cosine of the equated longitude
by the cosine of the equated latitude, and we divide the product
6 by the total sine, the quotient will be the sine of HL, which is
equal to the cosine of the distance HE. Further, the ratio of the
sine of HE to the sine of HK is as the ratio of the sine of EL to
the sine of LA. Hence, if we multiply the sine of the equated
longitude by the total sine, and we divide the product by the sine
of (MS 227) the distance (between the two towns), the quotient
9 is the sine of the distance of the azimuth (point L) from the
meridian, on the side of the town whose azimuth is required,
from the meridian of the other. It is either on the east side or the
west side, which is indicated to us by the magnitude of the longi-
12 tude. Again, the ratio of the sine of HB to the sine of BL is as the
ratio of the sine of HE to the sine of EK. Hence, if we like, we
multiply the cosine of the equated longitude by the sine of the
difference between the latitude of the town and the equated
latitude, and we divide the product by the sine of the distance.
The quotient obtained thereby is the sine of the distance of the
15 azimuth (point L) from the east-west line. It is either on the
south side, or on the north side, which is indicated to us by the
equated latitude. Further, there is an intersection of the two
horizons at the head of a quadrant from point L. Because H and
E are the poles of the two horizons, and since the circle EHL

18 passes through their four poles, the intercept on this circle included between the two (poles) is the maximum inclination of one horizon to the other, and it is a measure of the angle between them. Therefore, the intersection takes place at a complete quadrant from it.

But, if there is no difference in longitude between them
21 and there is only a difference in latitude, then the azimuth (point L) is on the meridian. Further, if the town whose azimuth is

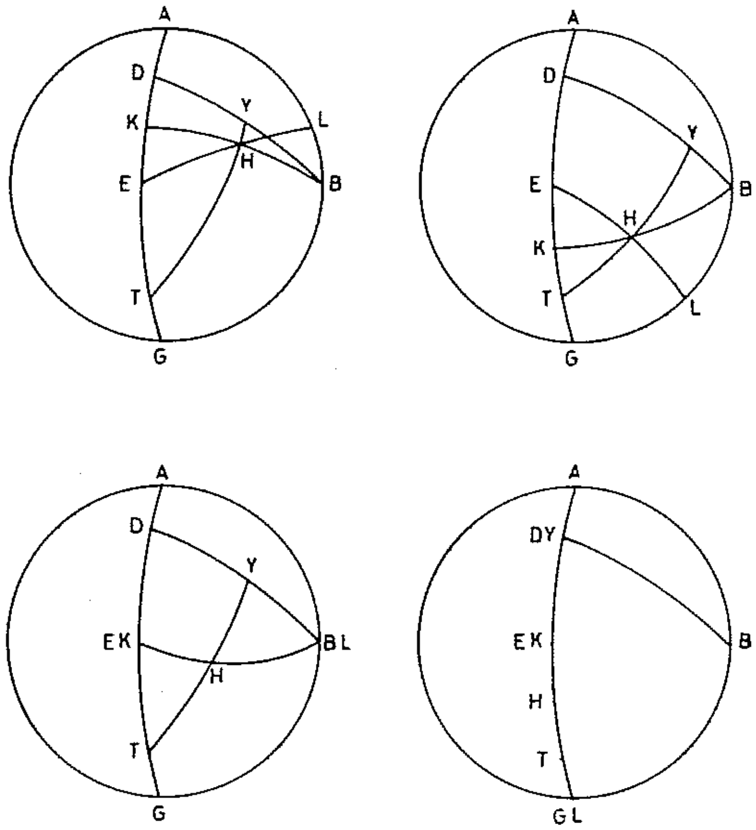

Figure 52

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required has the smaller latitude, it (point L) falls towards the south, but if it has the greater latitude, it falls towards the north. Also, the difference between the two latitudes is equal to the distance between the two (towns). (MS 228)

3 We mention the use of this art by travellers in following the courses to their destinations, and in reverting to those courses in cases of deviation from them, and, in particular, by desert wanderers at night who plan to raid their enemies at dawn, and by those who want to save their lives by escaping from their pursuing enemies. However, its more important use is in places of
6 worship, where prayers are held by young and old people of the faith (Muslims) (MS 229) and by those who have a divine book, or by non-Muslims living in Islamic lands. If at any town, H is replaced by Mecca, then the direction of the latter, relative to the former, would give the direction of Mecca. For it is a town of
9 known latitude and, according to various reports, its latitude has minutes in the twenty-second degree, though the calculators consider it to be twenty-one degrees. It is said that Manṣūr bin Ṭalḥa al-Ṭāhirī

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was ordered to find its correct magnitude. He found it to be in excess of that stated above by two thirds of a degree, which is in agreement with that reported by Ḥabash about the observation (for it ordered) by al-Ma'mūn. But others have claimed that the
3 increment is only one third of a degree. It is also a town of known longitude, for it is also told that Manṣūr bin Ṭalḥa found its longitude to be sixty-seven degrees, which is in agreement with what Ḥabash al-Ḥāsib has mentioned in his **Kitab al-abᶜād wal-ajrām.** He said that al-Ma'mūn had ordered someone to
6 observe lunar eclipses at it (Mecca), and that he had found a difference of three degrees between its meridian and that of Baghdad. So, if the longitude of Baghdad is seventy degrees, that of Mecca would be sixty-seven degrees.

9 We look around and we see that man's efforts are directed only towards earning a living, and for this purpose he endures hardships and fears, though he needs his food only once or twice

a day for his life in this world. But he pretends ignorance (MS 230) and neglects what he must not fail to do for his soul in the hereafter, five times in every day and night, thinking that his ignorance is a valid excuse, though he has the opportunity and the power to know it (what is good for his soul).

The Jews also need a direction, because they turn in their prayers to the Temple in Jerusalem which is of known longitude and latitude. It was also used as a symbol for eighteen months in the early days of Islam at Medina by the Prophet's followers who would turn on their heels (from the faith).

The Christians need the (direction of) true east because their elders, whom they call fathers, prescribed to them that they should turn to Paradise in their prayers. They added to that a proposition which is valid for them, that Paradise is in the orient of the world, and they deduced from it that prayers should be held in

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the direction of the middle of the orient, because it is the most probable, and, in general, the best alternatives of cases are the middle ones.

As to the arc of the distance, it is expressed in that unit which makes the measure of a great circle on a sphere three hundred and sixty degrees. Since the earth is at the center of the universe, and and its arcs are similar to those of the celestial sphere, a distance on the surface of the earth is also expressed in degrees, which make the measure of a great circle on the surface of the earth three hundred and sixty degrees. But that (degree) is unknown in terms of standards of lengths, which have been adopted by surveyors, such as: spans, cubits, fathoms, miles, (MS 231) and farsakhs. If the «share» of one degree is known, in any one of those standards, the circumference of the earth and other dimensions associated with it, as well as fractions thereof, will also be known. Again, if the distance between two given points is surveyed, and if the ratio of the arc between them to the circumference is also known, then the share of one degree and that of the whole (circumference) will be known.

It has been transmitted in books that the ancients found

that two towns, Raqqa and Tadmur (Palmyra), are on the same meridian, and that the distance between them is ninety miles. From these data, the share of one degree was known to be sixty-six miles and two thirds of one mile. This implies necessarily that the latitudinal difference between them is 1;21°. We said previously that the latitude of Raqqa is 36;1°. Therefore the latitude of Tadmur (Palmyra) is 37;22°. But this narrative is confused, because what is mentioned in it about the latitudes of the two towns does not give the appropriate magnitudes. It is probable that the manuscripts are corrupt, and since I have little confidence in this narrative, I have not evaluated the circumference. This story was told by Muḥammad bin ʿAlī al-Makkī in his book, **Fī al-ḥujja ʿalā istidārat al-samā' wal-arḍ.** (The Proof for the Rotundity of the Heavens and the Earth), and he claimed that the latitude of Palmyra is thirty-four, and that of Raqqa thirty-five degrees and one third of a degree.

However, al-Fazārī has mentioned in his zīj that the Indians consider the circumference of the earth to be six

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thousand six hundred farsakhs, where a farsakh is sixteen thousand cubits. He mentioned also that (the legendary Egyptian sage) Hermes considered it to be nine (MS 232) thousand farsakhs, where a farsakh is twelve thousand cubits. So the share of one degree out of three hundred and sixty degrees, according to the Indians, is eighteen farsakhs and one third of a farsakh. If each of them is three miles, then one degree would be equal to fifty- five miles, where each mile is five thousand three hundred and thirty-three cubits and one third of a cubit. But, according to Hermes, it would be twenty-five farsakhs which are equal to seventy-five miles, where each mile is four thousand cubits.

Then al-Fazārī claimed that some sages estimated each degree to be equivalent to one hundred miles, and hence their estimate of the circumference of the earth would be twelve thousand farsakhs.

Abū al-Faḍl al-Hirawī has mentioned, in (his book) **al-Madkhal al-ṣāḥibī,** that the last expedition of astronomers in al-Ma'mūn's days made their observations between the City of

Peace (Baghdad) and Samarra, because both are under one and
the same meridian circle, with a latitudinal difference of one
15 degree, and that they found that a degree of the meridian is
equivalent to 56;40 terrestrial miles, where one mile is equal to
four thousand **sawdā'** cubits.

I think that Abū al-Faḍl was careless in (reporting) this and
had not corroborated the evidence for it. He has not given

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us a report about the geodetic survey, which can be confirmed
from other sources, because it is unanimously recognized that
(MS 233) the latitude of Samarra is 34;12°, and that of Baghdad
3 is 33° plus some extra minutes, either 20 or 25. In (his) book,
al-Abᶜād, Ḥabash mentioned that he worked on those extra
minutes, and stated that the latitudinal difference between the
two cities is either 0;52° or 0;47°. Thus there is a difference
between these estimates and that of one degree, and if the
6 corresponding difference in miles is multiplied by three hundred
and sixty, the amount of the circumference corresponding to
these estimates would be too low, while that corresponding to a
difference of one degree would be excessive. Also, both cities are
on the banks of the Tigris, whose course does not run from north
to south in the direction of the same meridian line; it follows a
9 zigzag course, which runs from west to east. Moreover, the
number of farsakhs between the two cities, if we count them one
after another, is twenty-two, which is equal to sixty-six miles.
So how is it that the distance was found to be fifty-six miles and
two thirds of a mile!

12 As to the observations that were made by al-Ma'mūn
('s astronomers), they were started after he had read in some
Greek books that one degree of the meridian is equivalent to five
hundred stadia, where a stadium is the standard measure of
length which was used by the Greeks for measuring distances.
However, he found that its actual length was not satisfactorily
known to the translators, to enable them to identify it with local
15 standards of length. Then, according to Ḥabash, who obtained
his information from Khālid al-Marwarūdhī, al-Ma'mūn ordered
a group of learned astronomers, and expert (MS 234) carpenters

and workers in brass, to prepare the required instruments and to
select a locality for a geodetic survey. They chose a spot in the
plain of Sinjār, which is in the neighborhood of Mosul, nineteen
18 farsakhs from the town itself, and forty-three farsakhs from
Samarra. They liked its level ground, and transported their
instruments to it. They chose a site and observed with their
instruments the sun's meridian altitude. Then they departed
in two parties:

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Khālid, with the first party of surveyors and artisans, headed in
the direction of the north pole, and ᶜAlī bin ᶜĪsā, the maker of
astrolabes, and Aḥmad bin al-Buḥtarī, the surveyor, with the
3 second party, headed in the direction of the south pole. Each
party observed the meridian altitude of the sun until they found
that the change in its meridian altitude had amounted to one
degree, apart from the change due to variation in the declination.
While proceeding on their paths, they measured the distances
they had traversed, and planted arrows at different stages of their
6 paths (to mark their courses). While on their way back, they
verified, by a second survey, their former estimates of the lengths
of the courses they had followed, until both parties met at the
place whence they had departed. They found that one degree of a
terrestrial meridian is equivalent to fifty-six miles. He (Ḥabash)
claimed that he had heard Khālid dictating that number to Judge
Yaḥyā bin Aktham. So he heard of that achievement from Khālid
9 himself. Also, a similar narrative was told by Abū Ḥāmid al-
Ṣaghānī, who obtained his information from Thābit bin Qurra.
But it is said that al-Farghānī has reported an extra two thirds
of a mile, in addition to the (above -) mentioned number of miles.
(MS 235)

I have found all the other narratives in agreement about
12 these two thirds. However, I can not ascribe that omission to an
oversight in the manuscript of the **Kitab al-abᶜād wal-ajrām,**
because Ḥabash has derived on that basis the circumference of
the earth, its diameter, and all other distances. When I examined
those, I found that they were based on that assumption of fifty-
15 six miles. So it is preferable to ascribe the difference in the two

narratives to the two parties (who participated independently in that expedition). That difference is a puzzle; it is an incentive for a fresh examination and observations. Who is prepared to help me in this (project)? It requires

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a strong command over a vast tract of land and extreme caution is needed from the dangerous treacheries of those spread over it. I once chose for this project the localities between Dahistān, in the vicinity of Jurjān and the land of the Ghuzz (Turks), but the findings were not encouraging, and then the patrons who financed the project lost interest in it.

I have put in this table (which follows) the shares of miles and degrees according to both traditions, that of Ḥabash, and that of al-Farghānī, to be available when needed. (The text in the conversion table is to thirds)

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CONVERSION TABLE FROM MILES INTO DEGREES

FARSAKHS	MILES	ḤABASH				AL-FARGHĀNĪ			
		Deg.	Min.	Sec.	Th.	Deg.	Min.	Sec.	Th.
0	1	0	1	4	17	0	1	3	32
	2	0	2	8	34	0	2	7	4
1	3	0	3	12	51	0	3	10	35
	4	0	4	17	9	0	4	14	7
	5	0	5	21	26	0	5	17	39
2	6	0	6	25	43	0	6	21	11
	7	0	7	30	0	0	7	24	42
	8	0	8	34	17	0	8	28	14
3	9	0	9	38	34	0	9	31	56
	10	0	10	42	51	0	10	35	18
	11	0	11	47	9	0	11	38	49
4	12	0	12	51	26	0	12	42	21
	13	0	13	55	43	0	13	45	53
	14	0	15	0	0	0	14	49	25
5	15	0	16	4	17	0	15	52	56
	16	0	17	8	34	0	16	56	28
	17	0	18	12	51	0	18	0	0
6	18	0	19	17	9	0	19	3	32
	19	0	20	21	26	0	20	7	3
	20	0	21	25	43	0	21	10	35
7	21	0	22	30	0	0	22	14	6
	22	0	23	34	17	0	23	17	39
	23	0	24	38	34	0	24	21	10
8	24	0	25	42	51	0	25	24	42
	25	0	26	47	9	0	26	28	14
	26	0	27	51	26	0	27	31	46
9	27	0	28	55	43	0	28	35	17
	28	0	30	0	0	0	29	38	49
	29	0	31	4	17	0	30	42	21
	30	0	32	8	34	0	31	45	53

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CONVERSION TABLE FROM MILES INTO DEGREES

FARSAKHS	MILES	ḤABASH				AL-FARGHĀNĪ			
		Deg.	Min.	Sec.	Th.	Deg.	Min.	Sec.	Th.
10	31	0	33	12	51	0	32	49	24
	32	0	34	17	9	0	33	52	56
	33	0	35	21	26	0	34	56	28
11	34	0	36	25	43	0	36	0	0
	35	0	37	30	0	0	37	3	31
	36	0	38	34	17	0	38	7	3
12	37	0	39	38	34	0	39	10	34
	38	0	40	42	51	0	40	14	7
	39	0	41	47	9	0	41	17	39
13	40	0	42	51	26	0	42	21	11
	41	0	43	55	43	0	43	24	42
	42	0	45	0	0	0	44	28	14
14	43	0	46	4	17	0	45	31	45
	4	0	47	8	34	0	46	35	18
	45	0	48	12	51	0	47	38	49
15	46	0	49	17	9	0	48	42	21
	47	0	50	21	26	0	49	45	52
	48	0	51	25	43	0	50	49	25
16	49	0	52	30	0	0	51	52	56
	50	0	53	34	17	0	52	56	28
	51	0	54	38	34	0	53	59	59
17	52	0	55	42	51	0	55	3	32
	53	0	56	47	9	0	56	7	3
	54	0	57	51	26	0	57	10	35
18	55	0	58	55	43	0	58	14	7
	56	1	0	0	0	0	59	17	39
	57	1	1	4	17	1	0	21	11
19	58	1	2	8	34	1	1	24	42
	59	1	3	12	51	1	2	28	14
	60	1	4	17	9	1	3	31	46

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In the third chapter of (his) book, the Geography, Ptolemy pointed out that if this (arc of a) circle (for a geodetic survey) is not a meridian circle, but it is that (arc of a) circle which is included between the meridians of two localities, of known latitudes and longitudes, and if we know the angle included between this circle and the meridian of the place from which we start, that is, the angle which is the measure of the displacement of the azimuth from the meridian, and if we keep to this azimuth and follow the same direction, and if that distance (between the two localities) is determined by an actual survey, then the complete circumference of the earth can be determined from them (these data).

For in the figure presented (in the article) on the determination of azimuth (Fig. 52), if YH and DE, the two latitudes of the two towns H and E, are known, and if DY, the longitudinal difference is known, and if the distance EH is determined by an actual survey, and if the azimuth angle AEL is known, then the distance EH, expressed in degrees, would be known. For the ratio of the sine of TH to the sine of HK is as the ratio of the sine of TY, the quadrant, to the sine of YD. Therefore HK is known. Also, the ratio of the sine of HK to the sine of HE is as the ratio of the sine of AL to the sine of LE. Therefore HE is known. But the ratio of HE to three hundred and sixty is as the ratio of the distance EH to the length of the circumference of the circle that surrounds the earth. However, if the azimuth angle is not known, but the course of travel is in (MS 239) a fixed direction, so that the path is in a straight line, then we do not need it (the azimuth angle). For when the two latitudes and the longitudinal difference are known, the arc EH can be found, as was shown above in (the article on) the determination of azimuth, which is connected with this problem.

Here is another method for the determination of the circumference of the earth. It does not require walking in deserts.

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We climb a high mountain close to the seashore, or close to a

large level desert. If we find that sea, or that desert, to be lying to the east whence the sun rises, or to the west wherein it sets, we observe it until half its disc is screened from our eyes, and we measure its angle of dip at that instant by a ring provided with an alidade, like the ring ABGD (Fig. 53). When the position of the alidade is along HZ, the amount of the dip is given by arc BZ, and its complement by arc ZG. If the level plane does not happen to lie in one of those two (above-) mentioned directions, we set the ring vertically downwards, and we look with one eye through the two sights of the alidade until we see the spot of contact between the sky and the earth. The alidade would then take up a position like the first one, and the line of sight would coincide with the line HEZT, the direction of the alidade. We join T to the center of the earth, which is point K. Then we survey the perpendicular (height) of the mountain, which is EL. We drop the perpendicular ZM. The two triangles EZM and EKT are similar, and the ratio of EZ, (MS 240) the total sine, to ZM, the cosine of the dip angle, is as the ratio of EK to KT. If we apply (the rule of) **invertendo et dividendo** (to the proportion), then the ratio of EZ to the difference between it and ZM, which is equal to the versed sine of arc BZ, is as the ratio of EK to the difference between EK and KT, which is EL. Hence EK is known. But EL is known. Therefore LK is known, in the units of length in which EL was surveyed. And if the radius of the earth is known, then its circumference would also be known.

Again, we draw LO tangent to the earth at point L. Since angle E is known, the ratio of EL to LO is as the ratio of the sine of angle EOL, the dip angle, to the sine of angle OEL, the complement of the dip angle. Hence LO is known, and it is equal to OT. And EO is known, therefore ET is known. But the ratio of ET to KT is as the ratio of the cosine of the dip angle to the sine of the dip angle. Therefore the sides of the triangles KTE are known.

220 (MS 241)

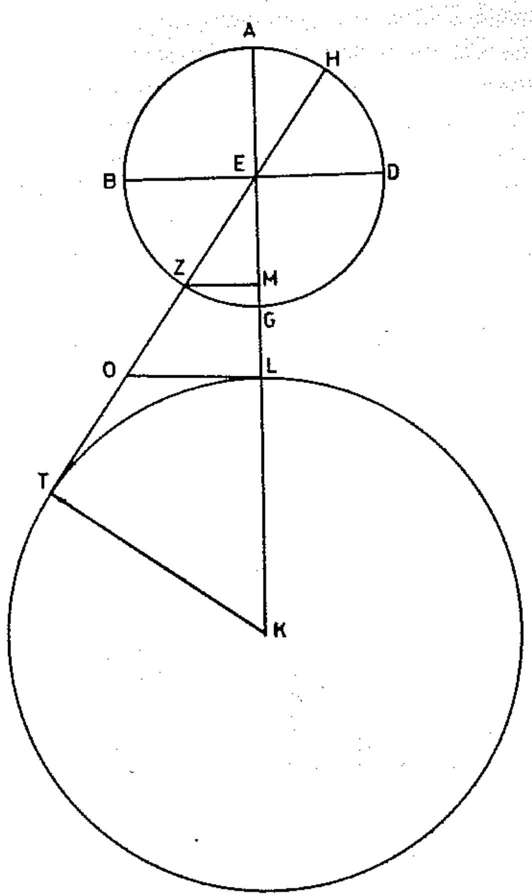

Figure 53

By this same method al-Ma'mūn ('s astronomer) has derived the circumference of the earth. Abū al-Ṭayyib Sanad bin ʿAlī has narrated that he was in the company of al-Ma'mūn when he made his campaign against the Byzantines, and that on his way he passed by a high mountain close to the sea. Then al-Ma'mūn summoned him to his presence and ordered him to climb that

mountain, and to measure at its summit the dip of the sun. He executed the order, and derived the circumference of the earth as follows: Let LT (Fig. 54) represent the circle of the earth, with center at K, LE the perpendicular (height) of the mountain, and LB (the tangent line at L) in the plane of the apparent horizon. We draw EZ, a tangent line to the earth, touching it at point T. Then arc BZ is a measure of the dip in the altitude circle. We join KT, and we drop BH perpendicular to EZ. Then BH is the sine of the dip, because M can be regarded as the center and MZ as the radius. Hence MH, the cosine of the dip, is known, and MB is the total sine. So the sides of the triangle BME are known, and it is similar to the triangle ETK. Therefore the ratio of MB to MH is as the ratio of EK to KT. Hence by (the rule of) **invertendo et dividendo,** the ratio of

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MB to the difference between MB and MH is equal to the ratio of EK to EL. So (EK is known and hence) LK is known, and this is what we wished to find. (MS 242)

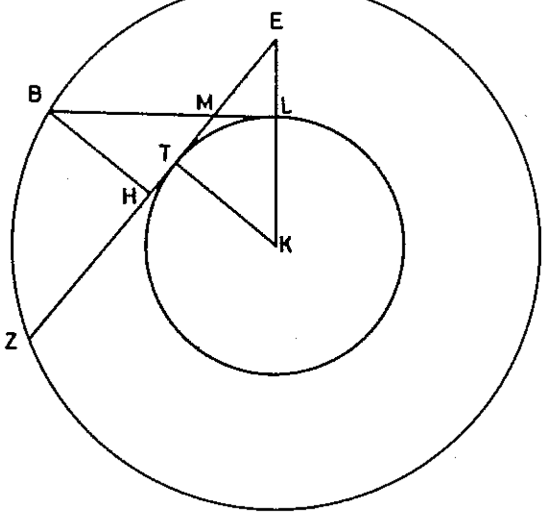

Figure 54

As to the determination of the perpendicular (height) of the mountain, it is made by one of the methods for the determination of distances. Let us construct for it a plate, with right-angled corners and of square shape, one cubit by one cubit, like the right-angled square ABGD (Fig. 55). We divide each of the two sides AB and AD into any number of parts we desire, provided that the divisions are equal in number and in magnitude. We fix at the two corners B and G two pegs normal to the surface of the square, and at corner D (G) we set an alidade provided with two vizors, or with a sharp edge and two pegs, whose length is equal to the diagonal of the square. Then let EZ represent the required perpendicular of the mountain, and ZG the plane of the horizon. We set the instrument perpendicular to it. Then we look from corner G, and we adjust the instrument by raising or lowering it until we see the tips of both pegs at G, and B, in line with (MS 243) the summit of the mountain, point E. We fix the instrument in that position. From D we drop a stone. Suppose that it falls at point H. We mark the distance between G and H, where the stone has fallen, in terms of the divisions on the side of the instrument. Then we turn to D; and we adjust the alidade by raising or lowering it, until

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we see the summit E, through the vizors, in line with the tips of the two pegs. This takes place as if the alidade were mounted at T. Since the triangles DAT and EGD are similar, the ratio of TA to AD is as the ratio of DG to GE. So we multiply the number of divisions in AD by the cubit DG, and we divide the product by the number of divisions in AT. The quotient obtained thereby is a measure of GE, expressed in cubits. But the ratio of GE to EZ is as the ratio of DG to GH, because the sum of the two angles DGH and EGZ is a right angle, and the sum of the two angles EGZ and GEZ is a right angle, and if we subtract the common angle EGZ, there remains angle DGH equal to angle GEZ and, therefore, angle GDH is equal to angle EGZ. So we multiply EG by GH, and we divide the product by the number of divisions in DG, the side of the square. The quotient obtained thereby is the required height, EZ. (MS 244)

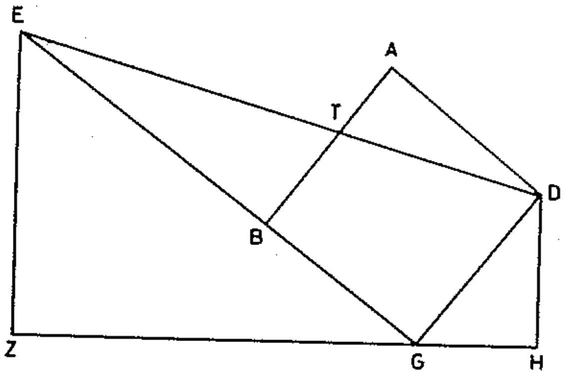

Figure 55

When I happened to be living in the fort of Nandana in the
land of India, I observed from an adjacent high mountain stand-
ing west of the fort, a large plain lying south of the mountain. It
12 occurred to me that I should examine this method there. So, from
the top of the mountain, I made an empirical measurement of the
contact between

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the earth and the blue sky. I found that the line of sight had
dipped below the reference line by the amount 0;34°. Then I
measured the perpendicular of the mountain and found it to be
3 652;3,18 cubits, where the cubit is a standard of length used in
that region for measuring cloth. Let it be represented in the
figure by EL (Fig. 56). Because angle T is a right angle, and
angle K is 0;34°, the dip angle, and angle E is 89;26°, the
complement of the dip angle, the angles of the triangle ETK are
6 known. Therefore its sides are known in the scale in which EK
represents the total sine. In this scale, TK is 59;59,49, and the
difference between it and the total sine is 0;0,11, which is the
measure of the perpendicular EL. But it is known in cubits, and
9 the ratio of its cubits to the cubits of LK is as the ratio of 0;0,11
to 59;59,49. The product of 652;3,18, the cubits of EL, times
59;59,49, the parts of LK, is 39,121;18,27,42. If it (the product)

12 is divided by 0;0,11, the parts of EL, the quotient would be
12,803,337;2,9 cubits, the radius of the earth. So its circum-
ference is 80,478,118;30,39, and the share (or equivalent) of
one degree out of three hundred (MS 245) and sixty is 223,550;
19,45 cubits. If this amount is divided by four thousand, the
15 quotient is 55;53,15 miles per degree, and that is not far from
Ḥabash's version. God is the true Helper!

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After that full discussion of what I have presented, my intention is the determination of the longitude of a specified city on the surface of the earth, whose position will be known,

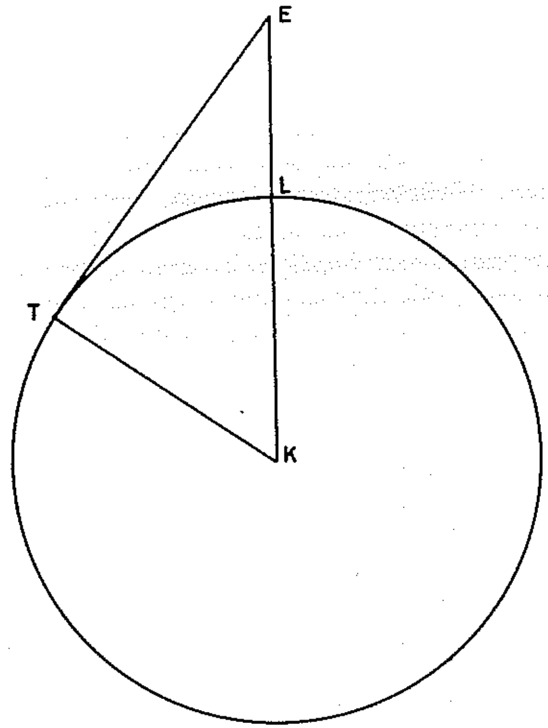

Figure 56

relative to all other cities. That city is Ghazna, but so far I have been able to determine its latitude only. As to its longitude, by the methods discussed above, there were reasons which prevented its determination. If I excuse myself for those reasons, my soul would feel ungrateful to God for the apparent and inner blessings He has bestowed upon me, and to my patron for the gifts and favors extended to me by his generous hand. However, I do beseech the Almighty to grant me the facilities for making the researches which (MS 246) I have loved, for in their pursuit I have not flinched from imminent danger to both body and soul. Nay, in those critical hours, I was always anxious to

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complete those researches before I pass away, and I requested Him to grant me a virtuous life here on earth, and in the hereafter.

I say: Most of the terrestrial longitudes and latitudes mentioned in (Ptolemy's) book (called) the **Geography** were derived on the basis of the reported distances between the various localities on the surface of the earth. Ptolemy himself certainly adopted the best methods, but others may or may not have followed him. However, determinations were based on hearsay evidence.

In the past, communications between those kingdoms were obstructed and risky. The divergence in the systems of beliefs adopted by their citizens was the main hindrance to travel between one kingdom and another. This is observed in the rush of a man of one faith to assassinate another of a different faith, as an act of devotion to his God. This is what the Jews do. Also it is observed in dragging him into servitude, the safest of what might befall him. This is what the Byzantines do. Or in looking down on him as a stranger, and in hurling accusations at him, which ultimately drive him to extremes of hardships and privations that agonize the soul.

But now Islam has appeared in the eastern and western parts of the world and has spread between Andalusia in the west and parts of China and central India in the east, and between Abyssinia and Nubia (MS 247) in the South and the Turks and the Slavs (**Ṣaqāliba**) in the North. It has united all the different

nations in one bond of love, a handiwork which can be made by God only. Fanaticism and racial prejudice have been swept away, and what remains is no more than the nuisance of corrupt and wicked highway robbers. The remaining nations, who have insisted on rejecting the faith, are afraid of the might of Islam; they highly respect its followers and conclude peace treaties with them. So, at the present day, hearsay evidence about the amounts of distances is more accurate and trustworthy. For in the book (called) the **Geography**, we often find places which are located to the east of others, but actually they are westerly, and vice versa.

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The reason for that is either the confusion in the reported distances from which the longitudes and the latitudes were derived, or the emigration of peoples from some towns to others and the transfer of the names of their former native towns to the latter ones. If Ptolemy allowed himself the use of that (method of distances), we too shall allow ourselves the use of it. But he, who has experienced the technicalities of observations, knows that the correction (of terrestrial longitudes) by the method of distances - when they are assessed carefully by distinguishing between level roads and rough mountainous ones, and by studying the nature of the slopes, the number of curves and the extent of their curvatures - if it is not superior to the correction obtained by observation of lunar eclipses, it is not inferior to it.

Let us now discuss methods for obtaining distances from given longitudes and latitudes, and those for obtaining longitudes and latitudes from given distances, in order to ascertain the relative positions of several famous cities, (MS 248) and finally realize our declared intention.

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VI. ON THE DETERMINATION OF DISTANCES, LONGITUDES, AND LATITUDES, FROM THE RELATIONSHIPS BETWEEN THEM

3 If two towns are on the same meridian, that is if the two longitudes are the same but the two latitudes are different, then the difference between the two latitudes is equal to the displacement between the two towns, measured on the circle. Hence, if it (the difference in latitude) is multiplied by the amount of
6 the surveyed degree mentioned heretofore, the product obtained is a measure of the distance between the two towns.

But if both towns are on the same parallel, that is, if the two latitudes are the same but the two longitudes are different, the displacement between them is the arc of the great circle
9 passing through both of them, and it is not the arc of the parallel of latitude between them. The chord of the displacement is the chord of the parallel of latitude between them and its ratio to the chord between the two longitudes is equal to the ratio of the cosine of their common latitude to the total sine. Hence, if we multiply the chord between the two longitudes by the cosine of their latitude and we divide the product by the total sine, the
12 quotient obtained is the chord of the displacement. If we multiply the displacement by the amount surveyed of one degree, the product obtained is a measure of the distance.

In case the two longitudes as well as the two latitudes are different, (MS 249) let A (Fig. 57) be one of the two towns, and B the other. We draw AB, the displacement arc between them.
15 Let E be the north pole of the equator, EAH the meridian of town A, and EBT the meridian of town B. With E as pole, we draw with (polar) distance EA the parallel of latitude AZ, and with distance EB the parallel BD. Then the points A, D, B, and Z are
18 concyclic, because the chords AD and BZ are equal, and the chords AZ and BD are parallel. But each of the two ratios, the sine of EA, the complement of the latitude, to the chord of AZ,

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and the sine of EB to the chord of BD, is as the ratio of the sine of EH, a quadrant, to the [chord] of HT, (the difference) between the two longitudes. Hence, if we multiply the cosine of each of
3 the two latitudes by the chord (of the difference) between the two longitudes, and we divide the product by the total sine, the quotient (in each case) is the chord (of the difference) between the two longitudes, in its (respective) parallel of latitude. And the sum of the product of chord AZ times chord BD plus the product of chord AD times the equal chord BZ is equal to the
6 product of chord AB times the equal chord ZD. So, if we multiply the quotients of the two divisions, the one by the other, and if we multiply the chord of the difference in latitude by itself, and if we add the two products, and then extract the (square) root of the sum, the root obtained thereby is a measure of the chord of the displacement AB. If we multiply the displacement
9 by the amount of the surveyed degree, the product will be a measure of the distance. (MS 250)

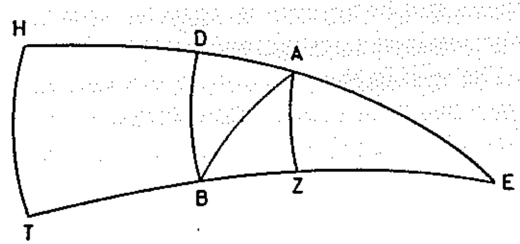

Figure 57

The Indians have a book on this subject, known as **Kitāb taḥdīd al-arḍ wal-falak.** First, its author derives half the town's parallel (**ṭawq al-madār**); he multiplies the versed sine
12 of the town's latitude by the number of farsakhs in the earth's semi-circumference, which is considered to be 3298 farsakhs and 17 out of 25 of a farsakh. Then he divides the product by 3438 minutes, and he subtracts the quotient from half a rotation, which

is 180. There remains half the parallel of that town. If the latitudes of the two towns are equal, he multiplies the difference in their longitudes by half the parallel, and then divides the product

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by 180. The quotient obtained thereby is expressed in grand farsakhs. Then he adds to the quotient one sixth of it, and he claims that the sum obtained is a measure of the distance, on the road used by humans and animals. If the two longitudes are equal, he multiplies the difference between the two latitudes by a quarter of the earth's circumference, which is 1649 farsakhs plus 17 out of 50 of a farsakh. (MS 251) Then he divides the product by 90, and there results grand farsakhs. He adds a quarter of it to get the measure of the road. That is what the author has claimed. If the two longitudes as well as the two latitudes are different, he obtains the displacement (between the two towns) by taking the difference between the two latitudes, then he squares the difference and retains the product. Further, he multiplies the longitude of each of the two towns by the respective half of the town's parallel and divides each product by 180. He takes the difference between the two quotients and he squares it; then he adds this quared difference to the retained number, and he extracts the (square) root of the sum. The unit in the root (which is a measure of the distance between the two towns) is a grand farsakh. He adds to the farsakhs of the root one third of their amount to get the measure of the road.

The object of that operation is to evaluate the **ṭawq al-madār,** which is half the circumference of the parallel of latitude, in the farsakh unit for a great circle, whose amount is 6597 farsakhs plus 9 out of 25 of a farsakh. Since the diameter of the earth, according to their tradition, is 2100 farsakhs, its circumference would be 6600 farsakhs, which is three plus one seventh times the diameter, according to the ratio which was derived by Archimedes.

But, in India, this ratio is the ratio of 3927 to 1250, because it was communicated to them, by divine revelation and angelic disclosure, that the circumference of the circle of the stars, that is, the circle of the zodiac, is 125,664,400 farsakhs, and that its

diameter is 40,000,000 farsakhs. So, according to this ratio, if the diameter of the earth is 2100 farsakhs, according to their hearsay evidence, the circumference of the earth would be (MS 252) 6597 farsakhs plus 9 out of 25 of a farsakh. As the authors of the **Little Sindhind** have dropped the noughts from the beginning (right) of the number of days given in the Great Sindhind and have dropped in it an

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equal number of noughts from the number of the sun's rotations, they have done so in this case. They have made the ratio of the diameter to the circumference equal to the ratio of 40,000 to 125,664, which was mentioned by al-Khwārizmī in his zīj, and in the **Kitāb al-jabr wal-muqābala,** after dividing each of those two numbers by two. But those two numbers have a common factor of thirty-two, therefore their ratio is what we stated above.

And I say: The ratio of one circumference to another circumference is equal to the ratio of the diameter of the former to the diameter of the latter. Similarly, for the semi-circumferences, the ratio of the radius of the parallel of latitude to the radius of the sphere is equal to the ratio of the semi-circumference of the parallel to the semi-circumference of a great circle. But if we regard the circumference as made up of 360 parts, the authors of the two **Sindhinds** regard it as made up of 114;36 (parts). Now, half this number is 57;18, and if it is expressed in minutes, its amount is 3438. Therefore, in their tables, they have put the total sine equal to this amount, and they have evaluated the other sines accordingly. Thus, in detail, the ratio of the radius of the sphere to the difference between it and the radius of the parallel, that is, to (MS 253) the versed sine of the parallel, is as the ratio of the semi-circumference of a great circle to the difference between it and that of the parallel. Therefore, if the versed sine of the town's latitude is multiplied by the semi-circumference of the earth and the product is divided by the total sine, the quotient is equal to the difference between the surveyed length of the semi-circumference of a great circle and that of the parallel. So when he (the author of the Sindhind) subtracted it (the quotient) from the semi-circumference of the

earth, there remained the **ṭawq al-madār,** I mean the number of farsakhs of the semi-circumference of the parallel of latitude.

Because the arcs of parallels intercepted between great
18 circles issuing from the pole are similar, if we suppose that the two towns of equal latitude, in the case mentioned above are represented (Fig. 58) by the points A and B, and if, with E as pole and distance

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EA we draw the parallel AS, then arc AB would be similar to arc HT. Hence the ratio of HT, the difference between the two longitudes, to the semi-circumference, which is 180, is as the
3 ratio of the number of farsakhs of AB to half the parallel. So, if the first is multiplied by the fourth and the product is divided by the second, the quotient is the third (AB). But AB, which is part of the parallel, is not the least distance between the two towns A and B; for the least distance between them is on the great
6 circle through A and B intersecting the parallel. Let it (the great circle) be KAS, and let M be the midpoint of arc HT. We draw the arc ESOM. Then the ratio of the sine of (MS 254) KA
9 to the sine of KS is as the ratio of the sine of AH to the sine of SM. But KA is part of KS, therefore AH is less than SM.

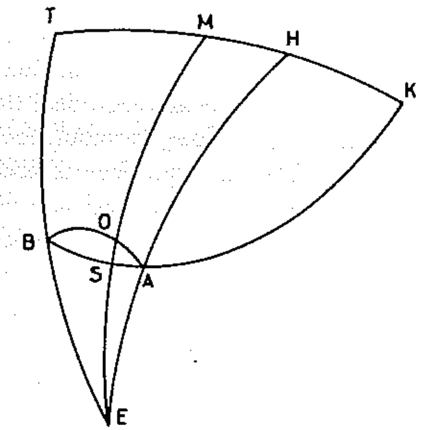

Figure 58

Also, HA is equal to MO, therefore MO is less than MS. But AS is the least distance between point A and the circle ESM; because
12 if a circle is drawn whose pole is A and whose (polar) distance is AS, it would touch the circle EM, and it would cut AO between the points A and O. Therefore AS is less than AO, and since ASB is twice AS, ASB is less than AOB. So their work in this case is not correct.

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As to the second case, where the two longitudes are equal but the two latitudes are different, their work in this case is
3 correct, because if B (Fig. 59) is on the meridian EAH, T is the center, GK is a quarter of the circumference of the earth, and if we draw ALT and BST, then the ratio of AB, the latitudinal difference, (MS 255) to EH, the quarter of the meridian, which is ninety degrees, is as the ratio of LS, the distance (between the two towns), to KG, which is a quarter of the circumference of
6 the earth. Hence, if the first is multiplied by the fourth, and the product is divided by the second, the quotient is a measure of LS.

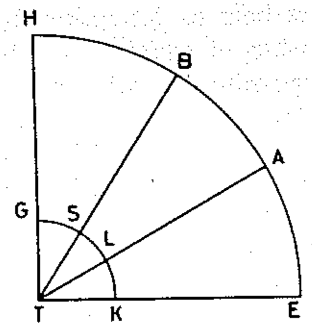

Figure 59

As to the third case, where the two latitudes as well as the two longitudes are different, the lack of precision and the carelessness in this case are beyond the tolerable limit. Let EHG
9 (Fig. 60) be the circle which represents the initial meridian at the

conventional eastern limit, or western limit, of the inhabited world. Then each of the arcs BZ and AD is a measure of the latitudinal distance. Upon my life, what I have stated is true. Also, GB is the longitude of town B, and HA is the longitude of town A. If these are converted from meridian degrees to farsakhs on the earth, the conversions from the meridians are valid, for they are simply the conversions of a set of numbers expressed in one unit to another set in another unit. (MS 256)

The author of the work assumed that the difference between GB and HA is equal to AZ. That assumption is false, for GB is similar to HZ, but it is not

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equal to it, and if he subtracts GB from HA, the remainder is greater than AZ. To find AZ he should start the work by taking the difference between the two longitudes. If he multiplies this difference by the semi-circumference of B's parallel and divides the product by one hundred and eighty, the quotient is the number of farsakhs in BD, and if he multiplies it by the semi-circumference of A's parallel (and divides the product by 180), the quotient is the number of farsakhs in AZ. Even when that is obtained, it does not help to determine the true value of AB, because the relationship of equality, which holds between the square of the side opposite the right angle and the sum of the squares of the two legs containing it, is a property of straight sides. But the sides of the triangle ABZ are made of arcs, and the magnitudes of its sides are not of a small order, to justify their treatment as straight lines.

If angle Z makes that equality hold because it is a right angle, angle D is also a right angle, and hence AB is the hypotenuse of the legs AZ and ZB, as well as the legs AD and DB. But AD is equal to BZ, therefore DB is equal to AZ. Since these arcs are similar, the ratio of DB to AZ is as the ratio of B's parallel to A's parallel. But the two parallels are different, and A's parallel is smaller than B's parallel. Hence AZ is smaller than DB. Therefore the basic assumption which led to their equality is absurd. (MS 257) The authors of this work have made the same assumptions, in this case and in the first case, which Marinos

made in his map of the earth, and which al-Battānī made in his determination of the direction of the **qibla.** They have treated the meridian circles as parallel straight lines, and the parallels of latitudes as parallel straight lines. Thus they have fallen into this outrageous error.

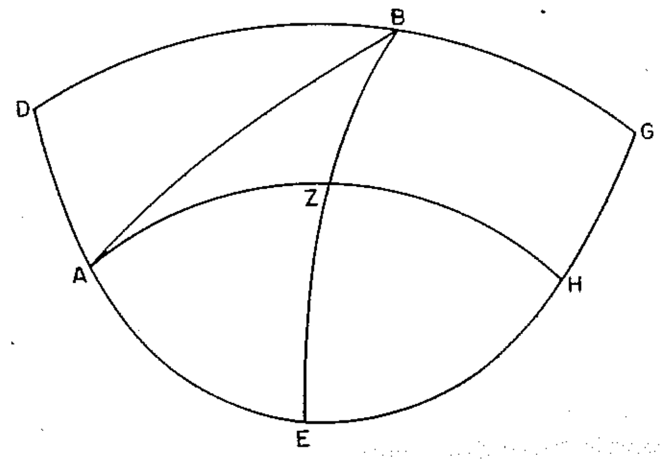

Figure 60

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As to the increments added to the calculated distances, they have been adopted because a distance is evaluated, if the correct method is used, along the path of the arrow. But roads are not like that, because there are windings to be followed, which make it necessary to deviate right or left, and up or down. Thus we know that the road traversed is (MS 258) necessarily longer than the calculated distance. By common consent, the calculators increase the distance by its sixth. It should not necessarily be so, because the amount of this increase depends on deviations whose number is not definitely known, and whose extents are also undetermined.

It is indeed surprising that the Indians augment the parallel of latitude by its sixth, the meridian circle by its fourth, and

the altitude circle by its third. I suppose they simply wanted to mention all the fractions in the work, because there is no universal cause which requires the application of this system in all circumstances for all countries.

12 Consider, for example, Mecca and Baghdad. The distance between them along the altitude circle is 12;1,51°, according to their longitudes and latitudes, where the latitude of Mecca is 21;40°, the latitude of Baghdad is 33;25°, and their longitudinal
15 difference is 3;0°. If we multiply it by the share for one degree in miles, the product is 681;44,50, which is a measure of the distance between them in miles. Al-Ma'mūn ordered someone to measure the length of the road. He found it to be 712 miles. The difference between the two measurements is 30;15 miles, which is approximately one third of one eighth of the total distance.

18 Then I say that between each pair of towns, these four things are interrelated: their two latitudes, their difference in longitude, and the distance between them. Whenever three of these are known, (MS 259) the fourth is deducible from them. The number of combinations is three. The first is made of the
21 two latitudes with the longitudinal difference. The distance is deducible from this combination, and this has already been discussed. The second is made of

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the two latitudes with the distance. The longitudinal difference is deducible from this combination. The third is made of the distance, the longitudinal difference, and one of the two latitudes.
3 The other latitude is deducible from it. These are the two main objects we have been pursuing in our study right from the beginning.

Now, let us evaluate the longitudes of some towns, or their latitudes, from the correct data known to us about one of them; or from data correctly derived for another town, so that we can derive the unknown data for the remaining towns. We make
6 Baghdad, the City of Peace, the reference base for measuring longitudes, because astronomical observations are made there, and it is the seat of the Caliphate and the source of royalty and princes. The difference (of longitude) between it and Alexandria

is known, because Baghdad is in the vicinity of Babylon; and Babylon was an ancient city long before the Deluge, and it existed
9 after it, down to the time of Alexander, in its present location.

As to the towns of known latitudes, which I take as reference bases in my worked examples, they are: Baghdad, Shīrāz, and
12 Sijistān, then Rayy, Nīshāpūr, and Jurjāniya in Khwārizm, and Balkh. Also other towns are added for corroborative evidence. If they do not follow their normal courses, I keep measuring one of them against another until (MS 260) my mind is fairly satisfied with the longitudes obtained and then I will gradually proceed to the intended base at Ghazna, because my observation and
15 operations are based at it. It is known that by taking them in pairs, some of them occupy extreme or mean positions, and that some are related to others in simple or complex relationships. The following examples will serve as guides to the calculator and as aids for formulation and investigation. I do not feel secure
18 against the occurrence of slips in the calculations, because of the intensity of my worries. God is the best Guide to the truth!

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VII. ON THE DETERMINATION OF THE LONGITUDINAL DIFFERENCE BETWEEN BAGHDAD AND RAYY

We mentioned before that the people of this profession
3 have accepted, by common consent, the rule to diminish the
distance by a sixth in such problems, because the geographical
distance follows the flight of the arrow. However, it does not
state the thing which determines this amount, nor the circum-
stance that leads to it; because distances vary with the even flat-
ness and the steepness (of the road), and also with the plurality of
6 curves and depressions or with their paucity. If the decrease is
due to these factors, then the amount of decrease should vary
with their variations, according to the impressions and estimates
of eyewitnesses of those variations, on the roads under consi-
deration. (MS 261) But, even when the roads are free from
9 ascents and descents, distances would be increased by such an
increment, if they run between mountains and through valleys,
because of the windings, and the interception of rivers the
crossings of which lengthen the road, or of muddy swamps which
have to be rounded by long circuitous roads, and by the compul-
12 sory deviation of the horse from the straight road towards the
source of water and the safe abode, both of which are indis-
pensable for travel in stages, and there are other similar causes.
Let A (Fig. 61) represent the position of Baghdad on the
surface of the earth or the zenith of its inhabitants on the celestial
sphere, and let AZ be part of its parallel of latitude, E the north
15 pole, and EDA its meridian. Then EA is its colatitude. Let B
represent Rayy, BD part of its parallel of latitude, and EBZ its
meridian. Then EB is its colatitude; AD is the latitudinal
difference, and AB, part of a great circle, is the

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distance between them. Though the distance between Baghdad
and Ḥulwān and that between Hamadān and Rayy covers

mountain passages, it requires a decrease of less than one sixth,
3 but that between Ḥulwān and Hamadān requires a decrease of
one sixth or more.
The number of farsakhs between Baghdad and Rayy is 158,
and by dropping its sixth, (there remains) approximately 132,
which is obtained by multiplying it (the 158) by five and dividing
the product by six. If the quotient is multiplied by three, it is
6 converted into 397 miles, (MS 262) and if this is divided by
56;40, according to the well-known opinion of the moderns,
which is not far from my own estimate mentioned above (223:14),
then the distance in degrees would be 7;0,21°.
9 Because the trapezoid whose sides are the chords AD, DB,
BZ, and ZA is cyclic, and because chord AD is equal to chord BZ,
and chord AZ is parallel to chord BD, the diagonals AB and ZD
12 are equal. The square of AB, the magnitude of the distance, is
equal to the square of the chord AD plus the product of chord
AZ times chord DB. But the ratio of chord AZ to chord DB is
equal to the ratio of the radius of the parallel AZ, which is the
sine of EA or the cosine of the latitude of Baghdad, to the radius
15 of the parallel DB, which is the sine of EB, the colatitude of Rayy.
As to the latitude of Baghdad, different observers have
found that it is neither less than 33;20° nor greater than 33;30°,
and the approved one is 33;25°,

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because it is also the mean between those two. As to the latitude
of Rayy, it was observed by Abū Maḥmūd al-Khujandī who
found it to be 35;34,39°, and this is in agreement with what Abū
3 al-Faḍl al-Hirawī had found in the days of Rukn al-Dawla.
Hence AD, the latitudinal difference between Rayy and Baghdad
is 2;9,39°. Its chord is 2;15,45, and the square of this is 5;7,8,3,
45. The chord of the displacement AB is 7;19,54, and the square
6 of this is 53;45,12,0,36. The difference between the two squares
is 48;38,3,56,51. We multiplied this difference by the cosine of
(MS 263) the latitude of Rayy, which is 48;47,59, and obtained
the product 2373;20,48,0,12,51,9. We divided this product by the
cosine of the latitude of Baghdad, which is 50;4,52, and obtained
9 the quotient 47;23,24,12,8. We extracted the (square) root of

this quotient and obtained 6;53,2. We multiplied this root by the
total sine and obtained the product 413;2,0; then we divided this
product by the cosine of the latitude of Rayy and obtained the
quotient 8;27,50, which is the measure of a chord whose arc is
12 8;5,20°, the longitudinal difference between Baghdad and Rayy.

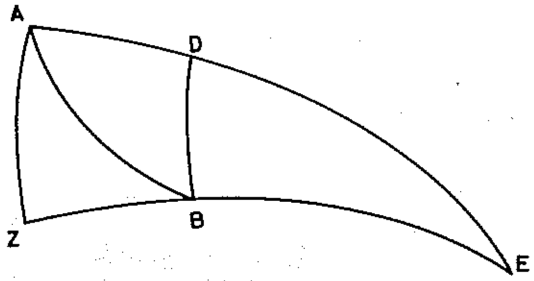

Figure 61

As to the amount used in the zījes, it is five degrees, but a comparison of the longitudinal differences between various towns does not confirm that current amount. The amount which we have derived is a good approximation to that which was mentioned by Abū Bakr Muḥammad

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bin Zakariya, the physician, in a treatise which he has written
on astronomy. He said that he had observed eclipses at Baghdad,
and that his brother had observed them at Rayy, and that he
derived from those two observations a difference of ten degrees
3 between the two cities. Although he is a trustworthy (MS 264)
and erudite gentleman, he may not have paid attention to the
various conditions discussed above, which must be applied to
observations made relative to the horizon, and because he has
not described the details of his observation, we do not have
complete confidence in his report.
6 Further, if we take the longitude of Baghdad to be 70
degrees from the coast of the Western Sea, the longitude of Rayy

would be 78;5,20°, and if we take the longitude of Baghdad to
be 80 degrees from the Immortal Isles, the longitude of Rayy
would be 88;5,20°. But what is intended in this chapter is the
9 evaluation of the difference in longitude between various towns;
it is not the evaluation of the longitudes themselves from the
beginning of the inhabited world. Therefore this difference of
opinion about an initial meridian for the measurement of longitude does not do us any harm, and our work in Khwārizm is a proof of the validity of this procedure.

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VIII. ON THE LONGITUDINAL DIFFERENCE BETWEEN JURJĀNIYA AND RAYY

I observed the latitude of Jurjāniya in the year four hundred seven of the Hijra, and found it to be 42;17°. Hence the latitudinal difference between it and Rayy is 6;42,21°. Its chord is 7;1,5, and its square (the chord's) is 49;15,11,10,25. The distance between the two (localities) is 185 farsakhs, but the road has many curves in the sands of the deserts, and many windings on the mountains and in the valleys. Therefore, no less than one sixth of the distance should be (MS 265) subtracted as we did in the case of the distance between Baghdad and Rayy. If we do that and we express the remainder in miles, the diminished distance would be approximately 463, and if this is expressed in degrees, it would be 8;10,14°. The chord of this is 8;33,16, and the square of this chord is 73;10,42,40,16. Hence the difference between the two squares is 23;55,31,29,51. We multiplied this difference by the cosine of the latitude of Jurjāniya, which is 44;23,22, and obtained 1062;2,9,19,23,29,42. We divided this product by the cosine of the latitude of Rayy, and obtained the quotient 21;45,47,21,30. We extracted the (square) root of this quotient and obtained 4;39,54. We multiplied this root by the total sine, and obtained 279;54,0; then we divided this product by the cosine of the latitude of Jurjāniya and obtained the quotient 6;18,20. It is a chord whose arc is 6;1,26°, the longitudinal difference between Rayy and Jurjāniya.

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IX. ON THE DETERMINATION OF LONGITUDE AND LATITUDE OF JURJĀN FROM THE LONGITUDES AND LATITUDES OF RAYY AND JURJĀNIYA

Let A (Fig. 62) represent the position of Jurjaniya, B the position of Rayy, and T the position of Jurjān, which lies on the road between them. We mentioned (in the last article) that the distance AB is 8;10,14°. Further, BT, the distance between Jurjān and Rayy, is seventy farsakhs, because the distance between them on the road through Qūmis, is 80 farsakhs, and on (MS 266) the road through Dunbāvand and the town of Sārya in Ṭabaristān, the distance is the same, as if the lengths of the two roads were approximately the same. But the distance via Āmul (in Ṭabaristān) is longer by ten farsakhs, and since Āmul and Sārya are equidistant from Rayy, the ten farsakhs are like the base of an isosceles triangle. Though the travel distance on the two roads between Rayy and Jurjān is the same, it is known that the road via Sārya is closer to a straight stretch, because there are more ascents and descents on it, and the same direction of travel is more strictly pursued. In fact, the linear path is between

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the road through Qūmis and that via Sārya, because the Qūmis road deviates to the north at Damaghān, and the Sārya road deviates to the east of Damaghān. However, travellers on the road between those two roads say that the distance is seventy farsakhs. By decreasing it by its sixth it becomes 175 miles, which is equivalent to 3;5,18°.

With T as pole and with the side of the square as radius, we draw the semi-horizon of Jurjān, and we extend EB in both directions to meet it at the two points Z and C. Also, we extend TB and TE to meet it at M and O respectively. Further, we construct THS perpendicular to BE. So the ratio of the sine of AB, the distance, to the sine of AE, the colatitude of Jurjān, is as the ratio of

the sine of angle BEA, the longitudinal difference between Rayy and Jurjāniya, to the sine of angle ABE. Hence, if we multiply the cosine of the latitude of Jurjāniya by the sine of the longitudinal difference between it and Rayy, which is 6;17,48, we get the product 279;30,19,55,36, and if we divide this product by the sine of the distance between them, which is 8;31,38, we get the quotient 32;46,41, which is the sine of angle ABE. But its ratio to the sine of the right angle THB is as the ratio of the sine of HT to the sine of TB. So, if we multiply the quotient of the (last) division by the sine of the distance between Rayy and Jurjān, which is 3;13,57, the product is 105;57,18,13, 57; and if we divide this product by the total sine, we get the quotient

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1;45,57, which is the sine of TH. Hence arc TH is 1;41,12°; its complement, HS, is 88;18,48°, and its sine, (the sine of HS), is 59;58,26. Further the ratio of the sine of BZ to the sine of BM, the cosine of BT, is as the ratio of the sine of the quadrant ZH to the sine of HS. But the complement of BT is 86;54,42°, and its sine is 59;54,46. So we multiply the sine of BM by the total sine, and obtain the product 3594;46,0. Then we divide this product by the sine of HS and obtain the quotient 59;56,20, which is the sine of BZ. Its arc is 87;24,57°, and its complement, BH, is 2;35,3°. Hence the difference between BH and the colatitude of Rayy is 51;50,18°, and this is the arc HE. The complement of this difference is 38;9,42°; it is arc EC, and its sine is 37;4,22. But its ratio to the sine of EO is as the ratio of the (MS 268) sine of CH to the sine of HS. So, if we multiply the sine of EC by the sine of HS, the product is 2223;23,55,9,32, and if we divide by the total sine, the quotient is 37;3,24, which is the sine of EO. Its arc is 38;8,33°, and this is the latitude of Jurjān, for it is the complement of its complement, I mean ET. So ET is 51;51,27°, and its sine is 47;11,19. But the ratio of the sine of ET to the sine of HT is as the ratio of the sine of EL, a quadrant, to the sine of LF. Hence, if we multiply the sine of HT by the total sine, the product is 105;57,0, and if we divide it by the sine of ET, which is the cosine of the latitude

of Jurjān, the quotient is 2;14,43. Its arc (sine) is 2;8,41°, which is the longitudinal difference between Rayy and Jurjān. Therefore the longitude of Jurjān is 80;14,1°, and this is close to what was related by Abū ʿAlī al-Sīnawī (Avicenna) in his treatise to Zarrayn Kīs, daughter of Shams al-Maʿālī. (He said)

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that he found it to be 79;20°, and he related also that he determined the latitude of Jurjān by observation of the fixed stars. On one occasion, he found it to be less than 37°, but on another, he found it to be more than that. So he regarded it to be necessarily 37°. However, Abū ʿAlī is not reliable, and at least he should not have been confused with the amounts of the altitudes of the (fixed) stars in his very complicated method for (MS 269) the determination of the longitude, or he should have adopted for his determination a method which does not depend on the observations of fixed stars by earlier observers. I suppose, if objections were raised against his method, he would put the blame for it on others.

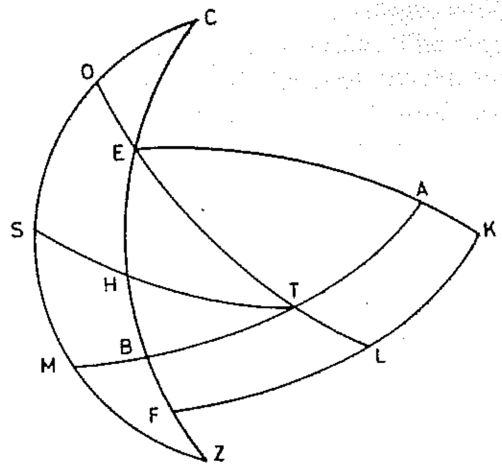

Figure 62

But Abū al-Faḍl al-Hirawī is fairly reliable, because of his competence in mathematics.

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He related that he had determined the latitude of Jurjān by observing the altitude of the vernal equinox. In the year three hundred seventy-one of the Hijra he found it to be 38°, but in the following (MS 270) year he found it to be 37° and two thirds of a degree. This confirms the reliability of our own derivation. The difference in the observations of the two years is due to the small size of his instrument, or to its lack of precision.

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X. CORROBORATION OF OUR RESULT FOR THE LONGITUDE OF JURJĀNIYA BY MEANS OF THE LONGITUDE OF THE CITY OF KHWĀRIZM

For this corroboration, I would present that I have observed, in the year three hundred eighty-five of the Hijra, the maximum declination at a village known as Būshkānz, one of the villages on the west bank of the Jayḥūn, and in the vicinity of the city of Khwārizm, and that I found the latitude of the village to be $41;36^\circ$. So the difference between this and the latitude of Jurjāniya is $0;41,0^\circ$. The chord of this difference is 0;42,56, and its square is 0;30,43,16,16. Further, the distance between this village and Jurjāniya is 17 long farsakhs, which is equivalent to 51 miles or to $0;54,0^\circ$. Its chord is 0;56,33, and its square is 0;53,17,54,9. So the difference between the two squares is 0;22,34,37,53. We multiplied this by the cosine of the latitude of Jurjāniya (MS 271) and obtained the product 16;42,11,20,5,12,26; then we divided this by the cosine of the latitude of the village, which is 44;52,4, and obtained the quotient 0;22,20,11,23. The (square) root of this is 0;36,36. We multiplied the root by the total sine, and obtained the product 36;36,0; then we divided this product by the cosine of the latitude of Jurjāniya and obtained the quotient 0;49,28. It is a chord whose arc is $0;47,14^\circ$. This arc is the longitudinal difference between Jurjāniya and the village of Būshkānz.

Further, let A (Fig. 63) represent Jurjāniya, B represent Būshkānz, and G the city of Khwārizm. The distance AB, as mentioned previously, is $0;54,0^\circ$; AG is 19 farsakhs, which is equivalent to 57 miles or to $1;0,21^\circ$, and BG is 3

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farsakhs, which is equivalent to 9 miles or to $0;9,32^\circ$. Let us, here and hereafter, call the distance AB and those like it and corresponding to it as follows: AB the first distance, AG the second distance, and BG the third distance.

Also, let KTW be part of the horizon of Jurjāniya, and TH part of the equator whose pole is E. With E as pole and with distance EB, we draw BD, the parallel of latitude which is the longitudinal difference between Jurjāniya and the village, measured along the parallel of the village. With A as pole and with distance A [D], we draw the almucantar DF. It is known that AD is the latitudinal difference between A and B, DN is the difference between AB and AD, and OG is the difference between AG and AB. But in this worked example it is 0;6,21 °; its chord is 0;6,39, and its square is 0;0,44,13,21. The chord of BG is (MS 272) 0;9, 59, and its square is 0;1,39,40,1. Hence the difference between the squares of BG and OG is 0;0,55,26,40. Now, according to what we presented above, if we multiply this difference by the sine of AB, which is 0;56,33, the product is 0;0,52,15,23,0, and if we divide the product by the sine of AG, which is 1;3,12, the quotient is 0;0,49,36,37. The root of this quotient is 0;7,2, which is the measure of chord OB. But its ratio to chord LC is as the ratio of the sine of AB to the sine of the quadrant AC. So if we multiply this root by the total sine, the product is 7;2,0, and if we divide this product by the sine of AB, the quotient is 6;40,36, which is a chord whose arc is 6;22,45 °, I mean the arc CL.

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Now we turn to the chord BD. We multiply the chord of the longitudinal difference between Jurjāniya and the village, the arc being 0;47,14 °, by the cosine of the latitude of the village; then we divide the product by the total sine, and obtain the quotient 0;36,51, which is the chord of the longitudinal difference, measured in the parallel of latitude of B. The square of this chord is 0;22,37,55,21. The difference between AB, the first distance, and AD, the difference between the two latitudes, is 0;13,0 °. Its chord is 0;13,37, and its square is 0;3,5,24,49. Hence the difference between the two squares is 0;19,32,30,32. We multiplied this difference by the sine of the latitudinal difference, which is 0;42,56, and obtained the product 0;13,58,59,42,53,52; then we divided this product by the sine of the first distance, which is (MS 273) 0;56,33, and obtained the quotient 0;14,50,10, 50 whose (square) root is 0;29,50. We multiplied the root by the

total sine, and obtained the product 29;50,0, then we divided this product by the sine of the latitudinal difference and obtained the quotient 41;39,36, which is the measure of a chord, whose arc is 40;37,42 °. This is the arc CK, and the sum of the arcs LC and CK is 47;0,27 °, whose sine is 43;54,12. The complement of KL, I mean LT, is 42;59,33 °, and its sine is 40;54,41. The ratio of the sine of TW, which is equal to KL, to the sine of WQ is as the ratio of the sine of angle Q, a right angle, to the sine of angle T. So, if we multiply the sine of KL by the cosine of the latitude of Jurjāniya, we obtain the product 1948;50,40,28,24, and if we divide this product by the total sine, we obtain 32;28,51, whose arc sine is 32;46,31 °. On subtracting this angle from ninety, there remains 57;13,29 °, which is arc QZ, the measure of angle M, whose sine is 50;26,53. If we multiply the sine of LT by the cosine of the latitude of Jurjāniya, the product is 1816;2,1,46,2; and if we divide this product by the sine of angle M, we obtain 35;59,53, whose arc (sine) is 36;51,3 °. This is the measure of arc LM, because the ratio of the sine of LT to the sine of LM is as the ratio of the sine of angle M to the sine of

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angle T. Further, GL is the complement of AG, the second distance, therefore GL is 88;59,39 °. So the difference between GL and LM, I mean MG, (MS 274) is 52;8,36 °, and the sine of this angle is 47;22,22. But its ratio to the sine of GS, where GS is the latitude of G, is as the ratio of the sine of angle S to the sine of angle M. Hence, if we multiply the sine of MG by the sine of angle M, the product is 2389;51,52,17,26, and if we divide this product by the total sine, the quotient will be 39;49,52, and its arc (sine) is 41;35,40 °, which is the latitude of the city of Khwārizm.

This result is in agreement with what I once had found in my youth; I think it was in the year three hundred eighty of the Hijra, or thereabout. I measured the meridian altitude at it (Khwārizm), at each of the two equinoxes, according to the computation in the zīj of Ḥabash al-Ḥāsib, by a ring which did not permit the measurement of fractions of degrees of a lower order than halves, and I found it to be 48;30 °. (MS 275)

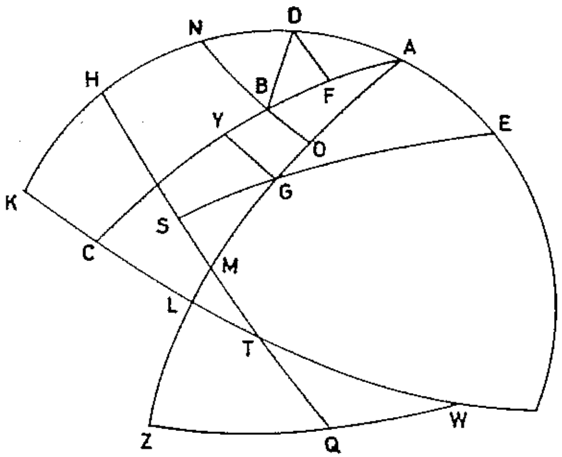

Figure 63

As to its longitude, we have said that the second distance, on a road east of the Jayḥūn, is 1;0,21°, that

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its chord is 1;3,11, and that the square of this is 1;6,32,8,1. Further, the latitudinal difference between the city (Khwārizm) and Jurjāniya is 0;41,20°; its chord is 0;43,17, and the square of
3 this is 0;31,13,26,49. So the difference between the two squares is 0;35,18,41,12. We multiplied this difference by the cosine of the latitude of Jurjāniya and obtained the product 26;7,27,19,26, 42,24, then we divided this by the [co] sine of the latitude of the city, which is 44;52,11, and obtained the quotient 0;34,56,0,
6 31, whose (square) root is 0;45,47,0. We multiplied it by the total sine, and obtained the product 45;47,0, then we divided this product by the cosine of the latitude of Jurjāniya, and obtained the quotient 1;1,53, which is the chord whose arc is 0;59,6°, the longitudinal difference between the city and Jurjāniya.

9 That evaluation tallies with what I had determined by observation. I had arranged with Abū al-Wafā' Muḥammad bin Muḥammad al-Buzjānī, he being in Baghdad and I in the city of

Khwārizm, for a joint observation by the two of us of a lunar eclipse which we observed in the year three hundred eighty-seven
12 of the Hijra. The difference in time between the two observations showed that there is a difference of approximately one equinoctial hour between their two meridians. Moreover, I observed several lunar eclipses, and the result in each case was close to this amount, differing from it only by an amount of insignificant order. (MS 276)

15 If we consider the longitude of the city to be 85;0°, then the longitude of Jurjāniya must consequently be 84;0,54°, because the latter is west of the former. We adopt this amount in our work, because the derived amount from our preceding problem was corroborated by observational evidence, and it gave
18 84;6,46°, as the longitude of Jurjāniya relative to Rayy, considering the distance between them. We now proceed from it to the city of Balkh.

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XI. ON THE DETERMINATION OF THE LONGITUDINAL DIFFERENCE BETWEEN JURJĀNIYA AND BALKH

The latitude of Balkh, according to an observation made by Sulaimān bin ʿIṣmat al-Samarqandī, in the two years, two hundred fifty-eight and (two hundred fifty-) nine of Yazdigerd, is 36;41,36°. So the latitudinal difference between it and Jurjāniya is 5;35,24°; its chord is 5;51,5, and the square of this is 34;14,19, 30,25. Further, the distance between them is 150 farsakhs, on a level road, where the estimated farsakhs are long ones and one of these is longer than the average farsakh. If we consider these as average farsakhs, the distance would be diminished, and because, from the precipice near the (fort) of Kālif up to Balkh, the direction of the road deviates from that of the first part of the journey, from Jurjāniya along the bank of the Jayḥūn, and comes closer and closer to the direction of the meridian line, (MS 277) the linear distance is, therefore, less than the distance covered. Hence we propose to drop out one third of one fifth of it, as a reasonable reduction. So the remaining distance is 140 farsakhs, which is equivalent to 420 miles, or 7;24,42°. The chord of this distance is 7;45,22, and its square is 60;9,26,8,4. So the difference between the two squares is 25;55,6,37,39. We multiplied this by the cosine of the latitude of Jurjāniya and obtained the product 1150;30,29,21,27,45,18; then we divided this product by the cosine of the latitude of Balkh, which is 48;6,38

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and obtained the quotient 23;54,49,49,34, whose (square) root is 4;53,24. We multiplied this root by the total sine, and obtained the product 293;24,0; then we divided this by the cosine of the latitude of Jurjāniya and obtained the quotient 6;36,35, which is the measure of a chord whose arc is 6;18,54°. This arc is the longitudinal difference between the two towns, and therefore the longitude of Balkh is 90;19,4 [8]°.

The validity of what is regarded as a basic rule must be substantiated by several operations, to satisfy the mind more convincingly with their available evidence. Since the modification of distances under consideration, by dropping out a fraction of them is an important matter and not a trivial one, and since it is an approximation to the true amount, on grounds of probability and not of certainty, I will, therefore, examine this work by considering the city of Bukhārā. For this purpose, I shall first determine the longitude of Darghān, and its latitude, because it is situated at the turn of the road to Bukhārā from the straight road (MS 278) that leads up to Balkh.

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XII. ON THE DETERMINATION OF THE LONGITUDE AND LATITUDE OF DARGHĀN FROM THE LONGITUDES AND LATITUDES OF JURJĀNIYA AND BALKH

Let us first evaluate the chord of the longitudinal difference between Jurjāniya and Balkh, in the parallel of Balkh, by dividing the difference between the squares of the chords AB and AD in the figure given previously (Fig. 61), I mean, by dividing the product of chord AZ times chord DB by the (square) root which gives the amount of chord AZ. The quotient is 5;18,1, which is the measure of chord BD, and its arc is 5;3,47°. But this arc, which we have taken, is not (measured in the parallel of) BD, because BD is one of the small circles; it is an arc of the great circle passing through the two points B and D, and its sine, which is 5;17,43, is a measure of the perpendicular from B to the diameter from D.

Further, let GDZ (Fig. 64) represent part of the horizon of Jurjāniya, GAZ half its meridian circle, ZTG the common line between their planes, TM the common line between this horizon and the circle which defines the distance between Jurjāniya and Balkh which is represented by B. Also, the distance AB is representing the complement of the altitude, whose sine TY is equal to 7;44,23. Further, YO is the sine of the longitudinal difference, in the parallel of Balkh; it is that amount (MS 279) which we derived in the introductory paragraph and found it to be 5;18,1. So the square of TY is 59;54,11,52,49, and the square of YO is 28;5,34,36,1. Hence the difference between the two squares is 31;48,37,16,48,

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and the (square) root of this, which is 5;38,24, is YL. But the ratio of TY to YL is as the ratio of TM to the sine of arc MD. So, if we multiply YL by TM, the total sine, the product is 338;24,0,

and if we divide this by TY, the quotient (giving the sine of MD) is 43;43,21. Hence arc MD is 46;46,42°; its complement GM is 43;13,18°, and its sine is 41;5,22.

Moreover, the ratio of TY to YO is as the ratio of TM to the sine of arc GM. So, if we multiply YO by TM, the total sine, the product is 318;1,0, and if we divide this by TY, the quotient is 41;5,[20], which is the sine of arc MG.

Also, let H represent Darghān; then AH represents the distance, which is 50 long farsakhs or 150 miles. It is covered on a level road with no significant winding. So it is sufficient to shorten its long unit and, consequently, its parts would be 2;38,49°. Its sine is 2;46,15, and it is represented by TN. But the ratio of this to FN is as the ratio of TM to the sine of MG. So if we multiply TN by the sine of MG, the product is 113;51,7,12,30, and if we divide this by TM, the total sine, the quotient is 1;53,51, which is NT (NF), the sine of arc HC in the parallel of Darghān. But the square of TN is 7;40,39,3,45; the square of NF is 3;36,1,49,21, (MS 280) and the difference between the two (squares) is 4;4,37,14,24. Its (square) root is 2;1,9, which is TF, the sine of AC. Its arc sine is 1;46,43°, and its sum with AE, the colatitude of Jurjāniya, is

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49;29,43°. But this is EC which is equal to EH, the colatitude of Darghān. Therefore the latitude of Darghān is 40;30,17°, and the

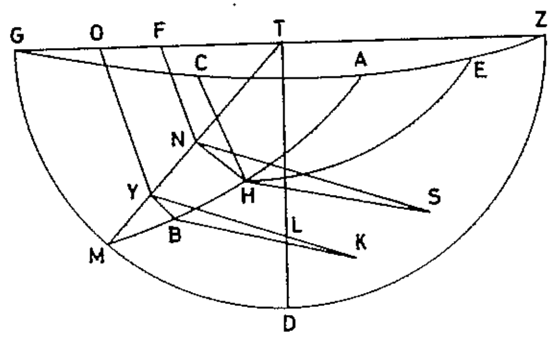

Figure 64

3 sine of EH, the sine of the colatitude of Darghān, is 45;37,17. But its ratio to the sine of HC, I mean NF, is as the ratio of the total sine to the sine of the longitudinal difference between Jurjāniya and Darghān. So, if we multiply NF by the total sine, the
6 product is 113;51,0; and if we divide this by the sine of EH, the quotient is 2;29,44, which is the sine of the longitudinal difference. The arc of this sine is 2;23,2°. Therefore the longitude of Darghān is 86;23,56°. (MS 281)

We shall also determine the position of Āmūya, which is
9 the ferrying point from Māwarā' al-Nahr (Transoxiana) to Khurāsān and ʿIrāq, in order to obtain from it, Darghān, and Bukhārā, a triangle whose vertices are situated at those three places and whose sides are the distances between them.

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XIII. ON THE DETERMINATION OF THE LONGITUDE AND LATITUDE OF ĀMŪYA FROM THE LONGITUDES AND LATITUDES OF BALKH AND JURJĀNIYA

3 We leave the representations in the (last) figure as they stand, and we do not change any of them, except point H. We suppose (here) that it represents Āmūya; then AH represents the distance of 105 (85?) grand farsakhs. On dropping out the five farsakhs and shortening its length, the distance would be 240
6 miles or 4;14,7°. Its sine is 4;25,52 and this is (represented by) TN. If we multiply TN by the sine of GM, the product is 182; 4,9,57,20, and if we divide this by the total sine, the quotient is
9 3;2,4, which is NF. Its square is 9;12,28,16,16; the square of TN is 19;38,5,5,4, and the difference between the two squares is 10;25,36,48,48, whose (square) root is 3;13,44, which is TF. Its arc sine, which is 3;5,6°, is the arc AC. But EH, the colatitude of
12 Āmūya, is equal to AC plus the colatitude of Jurjāniya. Therefore EH is (MS 282) 50;48,6°; its sine is 46;29,52, and the latitude of Āmūya is 39;11,54°. If we multiply NF by the total sine, the product is 182;4,0, and if we divide this by the sine of EH, the
15 quotient is 3;54,56. Its arc sine, 3;44,30°, is the longitudinal difference between Jurjāniya and Āmūya. Therefore the longitude of Āmūya is 87;45,24°.

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XIV. ON THE DETERMINATION OF THE LONGITUDE AND LATITUDE OF BUKHĀRĀ FROM THE LONGITUDES AND LATITUDES OF DARGHĀN AND ĀMŪYA

The distance between Darghān and Āmūya is 35 long farsakhs, on a straight road, and on diminishing it by one tenth, we make it 31 which is equal to 63 (sic) miles or 1;6,42°. We call it here the first displacement. The distance between Darghān and Bukhārā is 36 farsakhs. We reduce it similarly to 32, which is equal to 66 (sic) miles or 1;9,53°. We call it here the second displacement. The distance between Āmūya and Bukhārā is 20 farsakhs, and we make it 18 which is equal to 54 miles or 0;57,11°.

We apply these to the figure that we used in the determination of the latitude of the city of Khwārizm (Fig. 63). The difference between the first displacement and the second is 0;3,11°; its chord is 0;3,20, and its square is 0;0,11,6,40. The chord of the third displacement is (MS 283) 0;59,53, and its square is 1;1,26,30,49 (sic). Hence the difference between the two squares is 1;1,15,24,9. We multiply this by the sine of the second (first) displacement, which is 1;13,10 and we obtain the product 1;14,41,56,56,58,30; then we divide the product by the sine of the first displacement, which is 1;9,51, and we obtain the quotient 1;4,9,55,14, whose (square) root is 1;2,3. We multiply this root by the total sine and obtain the product 62;3,0; then we divide this by the sine of the second displacement and obtain the quotient 50;50,57, which is a chord whose arc is 50;8,33°. This is arc LC in that figure, and we designate it here as the first arc.

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The longitudinal difference between Darghān and Āmūya is 1;21,28°; the chord of this is 1;25,11, and the product of this times the cosine of the latitude of Āmūya is 66;0,50,8,32. We divide this product by the total sine and obtain the quotient 1;6,1, whose square is 1;12,38,12,1. Further, AD, the difference

between the latitudes of Āmūya and Darghān, is 1;18,23°, and the difference between this difference and the first displacement is 0;11,41°. The chord of this is 0;12,11, and its square is 0;2,28,26,1. Hence the difference between the two squares is 1;10,9,46,0. We multiply this by the sine of the difference in latitude between Āmūya and Darghān, which is 1;22,2, and we obtain the product 1;35,55,41,11,32,0; then we divide this by the sine of the first displacement and we obtain the quotient 1;22,24,2,24, whose (square) root is 1;10,19. We multiply this root by the total sine, and obtain the product 70;19,0; then we divide this by the sine of the difference between the latitudes of (MS 284) Āmūya and Darghān, and we obtain the quotient 51;25,49, which is the measure of a chord whose arc is 50;45,21°. This is arc KC in that figure, and we call it here the second arc.

The sum of the two arcs is 100;53,54°; its supplement is 79;6,6°, and its sine is 59;55,12. This is the sine of arc KL; its complement, 10;53,54°, is arc LT, and the sine of LT is 11;[20],39. We multiplied the sine of KL by the cosine of the latitude of Darghān and obtained the product 2733;29,24,54,32; then we divided this by the total sine, and obtained the quotient 45;33,29. Its arc sine is 49;24,1°, and the complement of this arc is 40;35,59°; it is the measure of angle M, and its sine is 39;2,46. We multiplied the sine of LT by the cosine of the latitude of Darghān and obtained the product 517;32,0,33,24; then we divided this by the sine of angle M and obtained the quotient 13;15,19. The arc of the quotient is 12;45,47°,

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and this is arc LM. Further, the complement of the second displacement is 88;50,7°, and the difference between this and arc LM is 76;4,20°, whose sine is 58;14,9. We multiplied this sine by the sine of angle M and obtained the product 2273;52,58,8,54; then we divided this product by the total sine and obtained the quotient 37;53,53. The arc of this (sine) is 39;10,15°, and this is the latitude of Bukhārā. The colatitude is 50;49,45°, and its sine is 46;30,57. Further, the difference between the latitudes of Bukhārā and Darghān is 1;20,[2]°; its chord is 1;23,49, and the square of this is 1;57,5,14,1. Also, the chord of the second dis-

placement is 1;13,10, and the square of this is (MS 285) 1;29,13,
21,40. Hence the difference between the two squares is 0;27,51,
9 52,21. We multiplied this by the cosine of the latitude of Darghān
and obtained the product 21;11,12,40,51,54,36; then we divided
this by the cosine of the latitude of Bukhārā and obtained the
quotient 0;27,19,42,52. We extracted the (square) root of this
and found it to be 0;40,29. We multiplied this root by the total
sine, and obtained the product 40;29,0. Then we divided this
12 product by the cosine of the latitude of Darghān and obtained
the quotient 0;53,15, which is the measure of a chord whose arc
is 0;50,11°. This arc is the longitudinal difference between
Darghān and Bukhārā, and hence the longitude of Bukhārā is
87;14,47°.
15 This work has, approximately, produced the current longitude, 87;30°, for Bukhārā, and its current latitude, 39;20°, and
we shall rely on it (this work), because it is confirmed by supporting evidence. Also, it gives us confidence in the validity of what
18 we have derived for Khwārizm, Darghān, and Āmūya. We
present now other evidence for the corroboration of the longitude
of Balkh.

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XV. ON THE DETERMINATION OF THE DISTANCE BETWEEN BUKHĀRĀ AND BALKH FROM THEIR LONGITUDES AND LATITUDES

The longitudinal difference between the two (cities),
3 according to our derivation of the longitude of Balkh, is 2;49,48°,
and the chord of this is 2;57,55. We multiplied this chord by the
cosine of the latitude of Bukhārā, and obtained the product
137;34,29,44,0. Then (MS 286) we divided this product by the
total sine, and obtained the quotient 2;17,34. Also, we multiplied
the chord of the longitudinal difference by the cosine of the lati-
6 tude of Balkh, and obtained the product 142;37,15,50,56; then
we divided this product by the total sine, and obtained the
quotient 2;22,37. We multiplied one of the two quotients by the
other and obtained the product 5;26,59,17,58. Further, the
difference between the two latitudes is 2;38,24°; its chord is
9 2;45,52, and the square of this chord is 7;38,31,45,4. The sum
of this square and the last product is 13;5,31,3,2; its (square)
root is 3;36,56, which is the measure of a chord whose arc is
3;27,11°. It is the arc of the distance between Bukhārā and
12 Balkh; we multiply it by 56;40 and obtain the product 195;40,23,
then we divide this product by three to convert it from miles into
farsakhs, and obtain the quotient 65;13,28.
The distance between Bukhārā and the Jayḥūn ferry at Kālif
15 is 55 farsakhs, and from Kālif to Balkh is 15. Thus the sum of
the two distances is 70, which is very close to what was derived
by computation. This corroboration shows that 90;19,48° is a
reliable estimate of the longitude of Balkh, but we shall reservedly
round off the fraction, though it is insignificant. We make the
longitude of

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Balkh 91°, because large errors are sometimes made in the
estimation of distances, and even the determinations of longitudes
by observations of eclipses are not free from such errors. There-

3 fore a set of determinations must be corroborated by evidence
from another set. (MS 287) Take Nīshāpūr, for example. It is
said that Manṣūr bin Ṭalḥa al-Ṭāhirī found its latitude by obser-
vation to be 36;10°. Also, Abū al-ᶜAbbās bin Ḥamdūn related
that he had determined, by observations of several eclipses, the
longitudinal difference between Baghdad and Nīshāpūr and
6 found it to be 12;30°. I think that this is reported in the **Kitāb
istadarat al-samā' wal-arḍ,** by Muḥammad bin ᶜAlī al-
Makkī, and that all its (Nīshāpūr's) astrologers work on that
basis. Further, it was found in the records of observations made
by the sons of Mūsā bin Shākir that an eclipse was observed at
Surra-man-ra'ā and Nīshāpūr, and that the longitudinal diffe-
9 rence was found to be ten degrees. Since surra-man-ra'ā is west of
Baghdad, the longitudinal difference between it (Baghdad) and
Nīshāpūr must be less than that mentioned. It is also said that
Manṣūr bin Ṭalḥa reported that he had found that difference,
and that it was found to be identical with that ascribed above to
12 Abū al-ᶜAbbās bin Ḥamdūn. It is preferable to rely on obser-
vations in case there is a striking similarity (in the narratives told
of an event), whether it is recorded permanently in a book by
Manṣūr or by other authors, because narratives, in general, are
confused and distorted. Moreover, confidence in an observer and
in his technical experience are essential conditions for a reliable
15 observation, because the determination of longitude, as we stated
before, may lead into error. Again, an observer's report must
inspire confidence in his work, and he must not suppress parts of
his findings, because this is one of the gravest accusations.
(MS 288) Further, the evidence about the distances between the
town (under consideration) and all the surrounding towns must
be thoroughly investigated.

18 It is possible that Manṣūr bin Ṭalḥa corrected that (latitude),
as much as he could, by theoretical considerations and not by an
actual observation, because he needed it for locating the positions
of the planets. Since he was interested in the science of the stars,

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his correction was thought to be based on an actual observation,
and, concerning Nīshāpūr, I have not found a reliable finding

ascribed to any other man. However, Manṣūr - a man of manifold
3 virtues - was more competent in physics and astrology than in
mathematics, and though he is trustworthy, he is not competent
enough in astronomy to lead researches in this field.

The evidence of available distances does not justify that
(correction). In particular, the story has also given a report
6 which makes it completely unreliable. It said: The longitudinal
difference between Mecca and Nīshāpūr was found to be 20;30°,
and that between Nīshāpūr and Balkh was found to be 10°.
However, the (longitudinal) difference from Mecca and the
9 aforesaid (longitudinal distance from Baghdad make the longi-
tudinal difference between Mecca and Baghdad necessarily 8°.
But it is known from a consideration of the distance between
the two, which is 712 miles, that the longitudinal difference is
less than that. Also, Ḥabash has stated in the **Kitāb al-Abᶜād
wal-Ajrām** that al-Ma'mūn ('s astronomers), by an observation
of a lunar eclipse, found it to be 3;0°. Therefore the first claim
is impossible.

12 Again, the distance between Balkh and Nīshāpūr, via
Baghshūr (MS 289) and Marv al-rūdh is approximately 80
farsakhs. But wherever this distance is taken, and in whatever
parallel it is laid, nay, in whatever part of the earth it is reckoned,
15 and in whatever configuration it is used, it can not produce what
they have stated (a longitudinal difference of 10°), unless it is
laid near the pole where distances become shorter for given
longitudinal differences.

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If we derive the longitudinal difference between Nīshāpūr
and Rayy by considering the distance between them unreduced
by one sixth or any other fraction, to be one hundred and thirty-
3 five farsakhs, then that difference would be 7;18,13°, and, upon
my life, the longitude of Nīshāpūr would be approximately that
longitude which is used by its astrologers, if the longitude of Rayy
is taken to be 85°, but, as stated before, the evidence of distances
rejects that conclusion.

6 If we compute the longitudinal difference between Nīshāpūr
and Balkh by considering the distance between them, reduced

approximately by its eighth, to be seventy farsakhs, we would find that difference to be 4;33,32°, and hence its (Nīshāpūr's) longitude, according to the longitude of Balkh, would be 86;26,28°. Again, if we derive it from (the longitude of) Jurjān, by considering the two of them and Jurjāniya to be situated at the vertices of a triangle, and taking the distance between Jurjān and Nīshāpūr, reduced by its tenth, to be 72 farsakhs, and the distance between Jurjāniya and Nīshāpūr, reduced by its sixth, to be 108 farsakhs, we would find the longitudinal difference between Jurjān and Nīshāpūr to be 4;31,56°, (MS 290) and hence the longitude of Nīshāpūr would be 84;45,57°.

Further, if we derive it from (the longitude of) Jurjāniya, by considering the two of them and Balkh to be situated at the vertices of a triangle, we would find the longitudinal difference between Jurjāniya and Nīshāpūr to be 1;56,58°, and hence the longitude of Nīshāpūr would be 85;57,52°. In all these derivations, it comes out larger than the current longitude. Also, in these problems, where three towns are used, the latitude of Nīshāpūr comes out larger than the amount we have mentioned for it. Therefore we turn southwards and take another direction to our destination.

XVI. ON THE DETERMINATION OF THE LONGITUDINAL DIFFERENCE BETWEEN BAGHDAD AND SHĪRĀZ

As to the distance between the two towns, it is one hundred and seventy farsakhs, and since most of it is covered on a level road, we reduce it by one tenth, by multiplying it by nine and dividing the product by ten. The quotient is

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153 farsakhs, which are equivalent to 459 miles or 8;6,0°. The chord of this arc is 8;28,32, and the square of this is 71;50,6,9,4. The latitude of Shīrāz, according to what was found by Ibn al-Ṣūfī, is 29;36°. Therefore the latitudinal difference between this latitude and that of Baghdad is 3;49°; its chord is 3;59,46, and the square of this is 15;58,8,3,16. (MS 291) So the difference between the two squares is 55;51,58,5,48. We multiply this difference by the cosine of the latitude of Baghdad and we get the product 2797;50,17,44,44,13,36; then we divide this product by the cosine of the latitude of Shīrāz, which is 52;10,10, and obtain the quotient 55;51,58,5,42, whose (square) root is 7;28,27. We multiply this root by the total sine and obtain the product 448;27,0; then we divide this by the cosine of the latitude of Baghdad, and obtain 8;57,16, a chord whose arc is 8;33,32°. This arc is the longitudinal difference between the two towns; and it is close to 9;0°, which is used by its astrologers. Hence the longitude of Shīrāz is 78;33,32°.

XVII. ON THE DETERMINATION OF THE LONGITUDINAL DIFFERENCE BETWEEN SHĪRĀZ AND ZARANJ, THE CITY OF SIJISTĀN

As to the latitude of Zaranj, it has come down to us that Abū al-Ḥasan Aḥmad bin Muḥammad bin Sulaimān had made an observation for it, with a quadrant whose diameter is twenty cubits,

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and found it to be 30;52°. All the astrologers in Zaranj use 31° because their instruments can not detect minutes accurately. The distance between Shīrāz and Sirjān, in Kirmān, is 78, and that from Sirjān to the tip of the plateau is 47, and that from this tip to Sijistān is 70. So the total distance is 195 farsakhs. We reduce this distance by one seventh, because the road is not rough all along, (MS 292) by multiplying it by six and dividing the product by seven. The quotient obtained is 168 farsakhs, which is equivalent to 504 miles, or 8;53,39°, whose chord is 9;18,16, and the square of this chord is 86;34,21,40,16. The difference between the latitudes of Shīrāz and Zaranj is 1;16,0°; its chord is 1;19,35, and the square of this is 1;45,33,30,25. So the difference between the two squares is 84;48,48,9,51. We multiplied this difference by the cosine of the latitude of Sijistān, which is 51;30,6, and obtained the product 4368;1,49,20,5,29,6. Then we divided this product by the cosine of the latitude of Shīrāz and obtained the quotient 83;43,39,54,42, whose root is 9;9,1. We multiplied this root by the total sine, and obtained the product 549;1,0. Then we divided this product by the cosine of the latitude of Sijistān and obtained the quotient 10;39,37, which is the measure of a chord whose arc is 10;11,36°. This arc is the longitudinal difference between the two towns, and hence the longitude of Sijistān is 88;45,8°. As a precautionary measure, we round off the fraction and consider the longitude of Sijistān to be 89;0°, which is close to the longitude of Balkh. For this reason, Sijistān was called

Nīmrūz, in relation to Balkh; it was also the seat of the Kiyāniān kings, and the native place of Magianism, their religion.

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If we derive the longitudinal difference between Nīshāpūr and Sijistān, we find it to be 4;12,16°, by taking the distance between them via Quhistān to be 120 farsakhs. Hence the longitude of Nīshāpūr would be 84;46,44°. This explains the tendency to regard the longitude of Nīshāpūr as 85;0°. Though we do not need this longitude for our plan, there was no harm in investigating it. Now, let us move on to our destination.

XVIII. ON THE DETERMINATION OF THE LONGITUDINAL DIFFERENCE BETWEEN BALKH AND GHAZNA

I found the (sun's) maximum altitude at it (Ghazna) to be 80;0°, in the year four hundred ten of the Hijra, by means of a quadrant whose diameter is nine cubits and whose circumference is divided into minutes, and in that year of the above-mentioned date, I found the minimum altitude to be 32;50°. Since half the difference between the two altitudes is 23;35°, this is the maximum declination, and hence the latitude of Ghazna according to this is 33;35°. The difference between this latitude and that of Balkh is 3;6,36°; its chord is 3;15,23, and the square of this chord is 10;36,14,38,49. The distance between the two (cities) is eighty farsakhs; we reduce one fifth of it and there remains 64, which is equivalent to 192 miles, or 3;23,18°. The chord of this arc is 3;32,52, and the square of this is 12;35,12,13,4. So the difference between the two squares is 1;58,57,34,15. We multiplied it by the cosine of the latitude of Ghazna, which is 49;59,5, and obtained the product 99;6,9,29, 43,36,15. We divided this product by the cosine of the latitude of Balkh, and obtained the quotient 2;3,35,37,4, (MS 294) whose (square) root is 1;26,4. We multiplied this root by the total sine and obtained the product [8]6;4,0;

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then we divided this product by the cosine of the latitude of [Ghazna] and obtained the quotient 1;43,21, which is the measure of a chord whose arc is 1;38,42°. This arc is the longitudinal difference between Ghazna and Balkh, and hence the longitude of Ghazna is 92;38,42°. We round off the fraction and make the longitude 93;0°, until we examine it from more reliable directions, because the distance between Balkh and Ghazna is not well defined nor truly estimated, and it is covered on a road with deep ravines, where the tributaries of the Jayḥūn

and the valleys of Ṭukhāristān and Khurāsān run northwards, where the valleys of Rakhaj and Zabulistān and some Indian rivers run southwards.

So let us investigate our required longitude from the direction of Sijistān, because the distance between them (Sijistān and Ghazna) is covered on a level road.

XIX. ON THE DETERMINATION OF THE LONGITUDINAL DIFFERENCE BETWEEN BUST AND SIJISTĀN

As to the latitude of Bust, I found that its inhabitants
12 assume for it 31;10°, but the operations which are discussed in the following chapters do not support that assumption. I found in Ghazna

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a zīj that was made according to the era of Diocletian, and was written on old parchment. At its end, there were comments made by some (MS 295) scholars, as well as jokes, birth dates,
3 and dates of solar eclipses which were observed between the years ninety and a hundred of the Hijra. Also, in the same handwriting, the latitude of Bust is given as 32;0°, and that the altitude of Capricorn was observed at it and was found to be 34;10°. It is known in that case that, when the least observed
6 altitude was found to be of that amount, the maximum declination, as reported by Ptolemy, was used in its derivation, and consequently the latitude was given as mentioned in the zīj, by dropping the one minute in the declination. If we compute in accordance with that altitude and the (maximum) declination of 23;35°, which we have found, and we add them, then the
9 sum 57;45° would be the colatitude of Bust, and therefore the latitude of Bust would be 32;15°. This is preferable to that used by its inhabitants because the following investigations corroborate it.

12 Some hate-monger, who prefers prejudice to justice, would imagine that my reliance on the old zīj, which I have cited, is analogous to what some would claim, concerning the story of Ptolemy's achievements in his book, the Tetrabiblos, that he had found it in an old copy of the Qur'ān, and that nothing else of
15 that copy has survived. But the zīj I have cited exists, and it is in the possession of Alī bin Muḥammad al-Wīshjardī who is nicknamed «The Spy of the Heavens».

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Let us now revert to the problem (MS 296) and say: the latitudinal difference between Bust and Sijistān is 1;23°; the chord of this arc is 1;26,55, and the square of this is 2;5,54,30,25.
3 The distance between the two towns is sixty farsakhs, and by dropping out one sixth of this distance, it is reduced to fifty, which is equivalent to 150 miles, or 2;38,49°, whose chord is 2;46,19, and the square of this is 7;41,1,14,1. So the difference
6 between the two squares is 5;35,6,43,36. We multiplied this by the cosine of the latitude of Sijistān and obtained the product 287;38,49,56,4,21,36. We divided the product by the cosine of the latitude of Bust and obtained the quotient 5;40,7,5,59, whose (square) root is 2;22,51. We multiplied this root by the total sine and obtained the product 142;51,0; then we divided this product
9 by the cosine of the latitude of Sijistān and obtained the quotient 2;46,25, which is the measure of a chord whose arc is 2;37,30°. Hence the longitude of Bust is 91;37,30°.

XX. ON THE DETERMINATION OF THE LONGITUDINAL
12 DIFFERENCE BETWEEN GHAZNA AND BUST

The latitudinal difference between the two towns is 1;20°. Its chord is 1;23,46, and the square of this is 1;56,56,51,16. The distance between them is 80, and by dropping one sixth of it, it is
15 reduced to 66 which is equivalent to 198 miles or 3;9,39°, whose chord is 3;18,38, and the square of this is 10;57,35,12,4. So the difference between the two squares is 9;0,38,20,48. We multiplied this difference by the cosine of the latitude of Bust, and obtained the product 457;13,58,[50],54,1,36. Then we divided it

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by the cosine of the latitude of Ghazna and obtained the quotient 9;8,50,50,[21], whose (square) root is 3;1,28. We multiplied this root by the total sine and obtained the product 181;28,0, then we
3 divided this by the cosine of the latitude of Bust and obtained the quotient 3;34,34, which is the measure of a chord whose arc is 3;24,56°. This arc is the longitudinal difference between the two towns, and hence the longitude of Ghazna is 95;2,26°. This result must be examined in various ways until one final amount is settled upon for the longitude.

6 ## XXI. ON THE DETERMINATION OF LONGITUDINAL DIFFERENCE BETWEEN GHAZNA AND SIJISTĀN

The latitudinal difference between the two towns is 2;43°.
9 Its chord is 2;50,41, and the square of this chord is 8;5,32,48,1. The distance between the two towns is 120, and by dropping out one sixth of it, it is reduced to 100 (farsakhs) which is equivalent to 300 miles or 5;17,39°, whose chord is 5;32,32 and its square is 30;42,58,25,4. So the difference between the two squares is 22;37,
12 25,37,3. We multiplied this difference by the cosine of the latitude of Sijistān and obtained the product 1165;9,45,2,38,12,18; then we divided this product by the cosine of the latitude of Ghazna and obtained the quotient (MS 297) 23;18,37,20,32, whose (square) root is 4;49,41. We multiplied this root by the total sine
15 and obtained the product 289;41,0; then we divided this product by the cosine of the latitude of Sijistān and obtained the quotient 5;37,29

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which is the measure of a chord whose arc is 5;22,24°. This arc is the longitudinal difference between the two towns, and hence the longitude of Ghazna is 94;22,24°.
3 I shall rely on this amount because it is close to the average between the smaller amount which we obtained relative to Balkh and the larger amount relative to Bust, and because the indirect method which follows this method produces an amount which is not far from that amount and corroborates it. For any one
6 determination, different amounts are obtained by direct methods and by their inverses, because of errors made by reporters of the distances, and because distances measured on steep roads are far from being level distances, and because errors are compounded, if long computations are made, which involve the use
9 of sines, chords, and irrational roots.

If the latitudes of Sijistān, Bust, and Ghazna have been

determined by observation, we can make Bust, which is at known distances from the other towns and lies between them, the town of unknown position, longitudinally and latitudinally, and derive its position by the method applied earlier in the case of Jurjān. This derivation will serve for an examination of the work and as a check on the computations. (MS 299)

XXII. ON THE DETERMINATION OF THE LONGITUDE AND LATITUDE OF BUST FROM THE LONGITUDES AND LATITUDES OF GHAZNA AND SIJISTĀN

We multiplied the cosine of the latitude of Ghazna by 5;37,7, the sine of 5;22,24°, the longitudinal difference between Ghazna and Sijistān, and obtained the product 280;50,40,58,35; then we divided this product by 5;32,10, the sine of 5;17,39°, the angular distance between them, and obtained the quotient 50;43,47. We multiplied this quotient by 2;46,15, the sine of 2;38,49°, the distance

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between Bust and Sijistān, and obtained the product 140;33,48, 58,45, which we regard as the first retained (number); then we divided this by the total sine and obtained the quotient 2;20,34. The arc (sine) of this quotient is 2;14,15°; its complement is 87;45,45°, and the sine of this is 59;57,15, which we regard as the second retained (number).

Further, we multiply 59;56,7, the sine of 87;21,11°, the complement of the angular distance between Bust and Sijistān, by the total sine and obtain the product 3596;7,0; then we divide this by the second retained number and obtain the quotient 59;58,51, whose arc (sine) is 88;33,25°. The complement of this is 1;26,35°; the difference between this complement and the colatitude of Sijistān is 57;41,25°, and the complement of this difference is 32;18,35°, whose sine is 32;14,11. We multiplied this sine by the second retained (number) and obtained the product 1932;42,20,59,45; (MS 300) then we divided this product by the total sine and obtained the quotient 32;12,42, whose arc (sine) is 32;28,13°. This arc is the latitude of Bust, and it is not far from that determined by observation. The colatitude is 57;31,47°, and its sine is 50;37,13. We divided the first retained number by this sine and obtained the quotient 2;46,37, whose

arc (sine) is 2;39,10°. This arc is the longitudinal difference between Bust and Sijistān. Hence the longitude of Bust is 91;39,10°, and the difference between this and that obtained relative to Sijistān alone is approximately two minutes. The mean of the two amounts is 91;38°, and we shall regard this as the longitude of Bust when we need it, if God permits.

XXIII. (QIBLA DETERMINATION)

(First Method): Though this determination of position is an end in itself, which satisfied an investigator, it is our duty to find an application for such a determination which is beneficial to the populace of the whole region whose longitude and latitude we have surveyed, or to a particular section of it exclusively. Let the universal benefit be the determination of the azimuth of the **qibla.** We discussed above the simple methods for that determination,

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which were used by our earlier people of the profession (astronomers). However, if we wish to find the azimuth of the **qibla** and more details are wanted, let it be known that we multiply the cosine of the latitude of our town by the sine of the longitudinal difference between our town and Mecca, and that we divide this product by the total sine, and obtain the quotient which is the sine of the (MS 301) «perpendicular». We find the arc of this sine and take its cosine, then we divide by this cosine the product of the sine of the latitude of our town times the total sine and obtained the quotient, which is the sine of an arc. We find the arc of this sine, then we take the difference between this arc and the latitude of Mecca. We multiply the cosine of this difference by the cosine of the perpendicular; then we divide this product by the total sine and obtain the quotient, which is the sine of an arc. We find this arc and take its cosine, then we divide by this cosine the product of the cosine of the latitude of Mecca times the sine of the difference between the two longitudes. The quotient obtained is the sine of the azimuth angle, measured from the meridian of our town. When we hold our prayers, we turn through that azimuth angle, east or west of the meridian, according to the position of Mecca relative to our town.

For example, in the case of the city of Ghazna, its longitude is west 94;22,24°, its latitude is north 33;35°, its colatitude is

56;25°, and the longitudinal difference between it and Mecca is 26;22,24°. We multiplied 49;59,5, the cosine of the latitude of Ghazna, by 27;35,14, the sine of the longitudinal difference and obtained the product 1378;56,22,42,10. Then we divided this product by the total sine, and obtained the quotient 22;58,56, which is the sine of the perpendicular whose arc is 22;31,19°. The complement of this arc is 67;28,41°, and its sine, which is 55;25,26, is the cosine of the perpendicular. The product of the sine of (MS 302) the latitude of Ghazna, which is 33;11,20, times the total sine, is equal to 1991;20,0. We divided this product by the cosine of the perpendicular and obtained the quotient 35;55,44, whose arc (sine) is 36;46,48°. The difference between this arc and the latitude of Mecca is 15;6,48°. The complement of this difference is 74;53,12°,

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and the sine of this is 57;55,29. We multiplied this sine by the cosine of the perpendicular and obtained the product 3210;24, 48,7,34, then we divided this by the total sine and obtained the quotient 53;30,25, whose arc (sine) is 63;5,54°. The complement of this arc is 26;54,6°, and its sine is 27;8,51. We divided by this sine the product of the cosine of the latitude of Mecca times the sine of the longitudinal difference, which is 1538;17, 11,24,6, and obtained the quotient 56;39,50, whose arc (sine) is 70;48,15°. This arc gives the direction of the **qibla** at Ghazna measured from its true south point on the horizon circle.

Proof of that (rule): We represent the horizon of Ghazna by circle ABG (Fig. 65) whose pole is E, and the circle of its meridian by AEG whose pole is the west point B, because Mecca is west of it (Ghazna). Let BH be a quadrant of the equator whose pole is T. We draw TL the circle of the meridian of Mecca, then arc HL is a measure of the longitudinal difference, and, on circle TL, we take the arc LM equal to the latitude of Mecca. Then M is the zenith of the people of Mecca. Through the two points E and M we draw a great circle; then this circle gives the boundary of the azimuth of the **qibla**. Let this circle intersect the horizon at point S, then S is (MS 303) the azimuth of the **qibla.** The displacement of S from point A, which is the

south point at Ghazna, is given by arc AS, and from the west point by arc SB.

We draw the circle of the meridian of Mecca, such that arc CMO is the part of that circle which falls above our horizon. With O as pole and with distance equal to the side of the square, we draw the (great) circle EDZ. Then this circle is orthogonal to circle CMO and to circle CSO. The ratio of the sine of TE, the complement of the latitude of Ghazna, to the sine of ED, the perpendicular, is as the sine of TH, the quadrant, to the sine of HL. Therefore, the unknown perpendicular ED, has been determined, and its complement ZD is also determined. Further, the ratio of the sine of OT, which is the complement of DT, to the sine of TG, which is

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the complement of ET, is as the ratio of the sine of the quadrant OD to the sine of DZ, the complement of the perpendicular. Hence arc OT is known. Each of the two arcs LT and DO is a quadrant; therefore, by dropping out the common arc between them, which is DT, there remains TO equal to DL. So MD, the difference between it (LD) and the latitude of Mecca, is known, and hence its complement MC is known. Also, the ratio of the sine of MC to the sine of MS, which is called the altitude of Mecca at the town, is as the ratio of the sine of the quadrant CD to the sine of ZD, which is the complement of the perpendicular. Therefore arc MS is known. Its complement, ME is also known, and is a measure of the distance between our town and Mecca. Further, the ratio of the sine of ME to the sine of MT, the complement of the latitude of Mecca, is as the ratio of the sine of angle ETM, (MS 304) the longitudinal difference, to the sine of angle TEM. Therefore angle TEM is known, but its sine is equal to the sine of its supplement, I mean the angle HES, and this angle is measured by arc AS, which gives the displacement of the direction of the **qibla** from the south point, and that is what we wanted to prove.

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Another Method: If we wish (to have another method), we

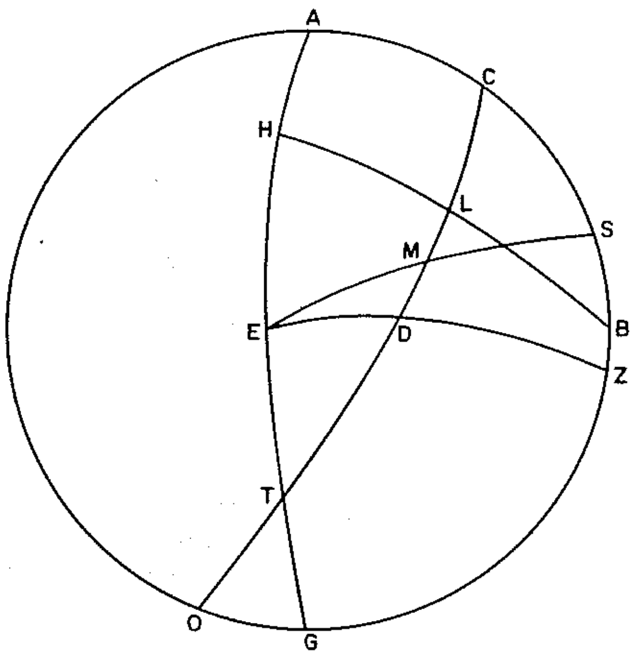

Figure 65

multiply the cosine of the difference in latitude between the latitude of our town and that of Mecca by the total sine, then we divide this product by the cosine of the latitude of our town and obtain the quotient which is a measure of the «diameter». Further, we multiply both the direct sine and the versed sine of the difference in longitude between the longitude of our town and that of Mecca by the (MS 305) cosine of the latitude of Mecca, then we divide each product separately by the total sine. In the case of the ordinary sine, the quotient is the sine of an arc which is called the modified (difference in) longitude, and in the case of the versed sine, we subtract the quotient from the «diameter», and we multiply the remainder by the sine of the latitude of the town, and then we divide this product by the total sine and retain this quotient.

Also, we multiply the sine of the latitude of Mecca by the

total sine, then we divide the product by the cosine of the latitude of our town. The quotient obtained thereby is the «gauge» by means of which we know on which side (of the east-west line) the azimuth lies. If the gauge is less than the retained, the azimuth is south of the east-west line; if it is equal to the retained, then the azimuth is coincident with that same line, and if it is larger than the retained, then the azimuth is north (of the east-west line). To find the amount of the azimuth, we square the difference between the retained and the gauge, and we square the sine of the modified longitude; then we divide the (square) root of the sum of the two squares by the product of the sine of the modified longitude times the total sine. The quotient obtained thereby is the sine of the displacement of the azimuth from the meridian. The gauge directs us whether the azimuth is north or south (of the east-west line), and the sense of the azimuth is east, or west, depending on the position of Mecca relative to our town.

For example, for the town of Ghazna whose longitude and latitude we have determined, the difference between its

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colatitude and the colatitude of Mecca is 11;55° (MS 306). The complement of this is 78;5°, and its sine is 58;42,25. We multiplied this sine by the total sine and obtained the product 3522;25,0; then we divided this by the cosine of the latitude of Ghazna and obtained the quotient 70;28,12, which is the measure of the diameter. As to the sine of the longitudinal difference, the ordinary sine is 27;35,14, and the versed sine is 6;43,9. We multiplied each of them by the cosine of the latitude of Mecca. The product for the ordinary sine is 1538;17,11,24,6, and that for the versed sine is 374;39,58,47,51. We divided each product by the total sine. The quotient from the ordinary sine is 25;38,17, which is the sine of the modified longitude, and the quotient from the versed sine is 6;14,40. Then we subtracted the last quotient from the diameter and obtained the remainder 64;13,32. We multiplied this remainder by the sine of the latitude of Ghazna and obtained the product 2131;34,29,22,40, then we divided this product by the total sine, and obtained the quotient 35;31,34, which is the retained (amount).

Further, we multiplied the sine of the latitude of Mecca by the total sine and obtained the product 1329;8,0; then we divided this by the cosine of the latitude of our town and obtained the quotient 26;35,27, which is the gauge. Since the gauge is less than the retained, we say that the azimuth of the **qibla** at Ghazna is inclined to the east-west line southwards. Further, we subtracted the gauge from the retained and obtained the remainder 8;56,7; then we squared this and obtained 79;50,21,4,49. (MS 307) Also, we squared the sine of the modified (difference in) longitude and obtained 657;18,35,36,49. We added the two squares and obtained the sum 737;8,56,41,38, and the (square) root of this sum is 27;9,1. We divided by this root the product of the sine of the modified longitude times the total sine and obtained the quotient 56;39,29. The arc (sine) of this quotient is 70;47,13 °, which is a measure of the displacement of the azimuth of the **qibla** at Ghazna from the south point, and the sense of the displacement is westwards.

Proof of this (method); We let the circle ABG (Fig. 66) represent the horizon of Ghazna, and we let AEG represent the line of intersection of its plane with the plane of its meridian, EB represent a segment of

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the line of intersection of its plane and the plane of the equator, TK a segment of the line of intersection between its plane and the plane of the parallel of Mecca, and HT a segment of the line of intersection between the plane of this parallel and the plane of the meridian of Ghazna.

We drop HD from the surface of the sphere, perpendicular to plane ABG. Then HDT is the triangle of daylight for the parallel of Mecca. Let point Z be the foot of the vertical drawn from Mecca to the plane of the horizon of Ghazna, and let us draw the line EZS. Then the direction of prayer coincides with this line, and arc AS is a measure of the displacement of the azimuth of the **qibla** from the south point. Then, at point Z, we erect ZM perpendicular to the plane of the horizon. So M, on the celestial sphere, is the zenith of the people of Mecca. Further, we draw ZC parallel to AG, and we join MC. Then

MZC is the triangle of time. Also, we draw MO parallel (MS 308) to BE. So it is equal to FE, the sine of the longitudinal difference, on the assumption that the radius of the parallel of Mecca is the total sine. Similarly, in this scale, HO is the versed sine of the longitudinal difference. If both are transformed into the scale where the radius of the parallel is equal to the cosine of its declination, then both will become the kind of sines that are measured in great circles.

It is known that HD is the sine of the meridian altitude of the parallel of Mecca, and hence it is equal to the cosine of the latitudinal difference between the two towns. But the ratio of HD to HT is as the ratio of the sine of angle HTD, which has the magnitude of the colatitude of Ghazna, to the sine of angle HDT, the right angle. Therefore HT, the diameter, is known, and hence HO, the transformed versed sine, is known. So the remainder TO is known, and it is equal to MC. Further, the ratio of MC to CZ is as the ratio of the sine of the right angle MZC to the sine of angle CMZ, which has the magnitude of the latitude of Ghazna, because the triangle MCZ is similar to the triangle HTD, and since angle MCZ is equal to the colatitude, its complement, CMZ, is equal to the latitude itself. Hence, CZ, the retained (amount), is known.

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We drop EL perpendicular to HT. Then EL is equal to the sine of the latitude of Mecca, because it is a segment of the axis between the center of the (celestial) sphere and the center of the parallel of Mecca. Moreover, the ratio of EL to ET, the sine of the rising amplitude of (MS 309) the parallel of Mecca, is as the ratio of the cosine of the latitude of Ghazna, I mean the sine of angle T, to the total sine, that is, to the sine of angle L. So ET, the gauge, is known, and therefore FZ, the difference between the gauge and the retained, is known. But the line segment ZE is the hypotenuse of a right triangle having FZ and FE as legs, FE being equal to MO, the sine of the modified longitude. Therefore ZE is known. Also, the ratio of ZE to FE is as the ratio of ES, the total sine, to the sine of arc AS. Hence the sine of arc AS is known, which is what we wanted to find.

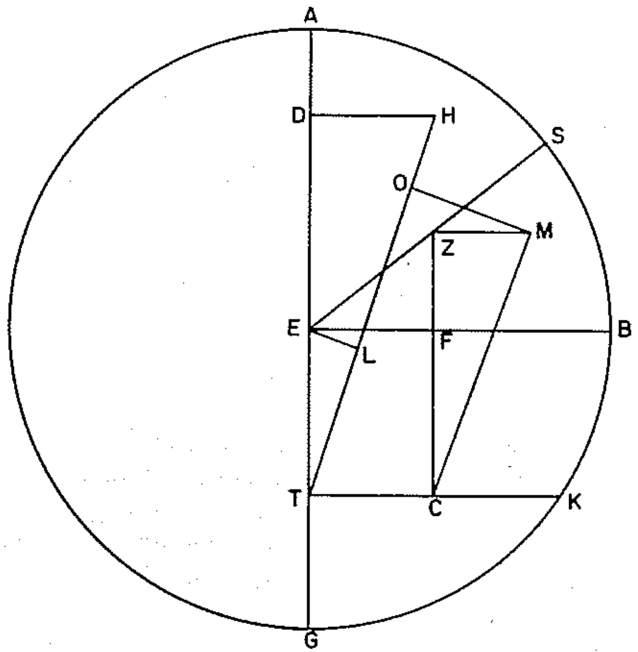

Figure 66

Third Method: We transform the direct sine and the versed sine of the longitudinal difference as we explained previously, (MS 310) in order to obtain from the ordinary sine the sine of the modified longitude. Further, we multiply the transformed versed sine by the sine of the latitude of our town, and we divide this product by the total sine, then we add the quotient obtained thereby to the versed sine of the sum of Mecca's latitude and the colatitude of our town. We obtain thereby the gauge. If this is less than the total sine, the azimuth of the **qibla** is

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south of the east-west line; if the gauge is equal to it, the azimuth coincides with that line, and if the gauge is more than the total sine, then the azimuth is north of the east-west line.

Further, we square the difference between the gauge and

the total sine, and we square the sine of the modified longitude; then we divide the product of the sine of the modified longitude times the total sine by the (square) root of the sum of the two squares. The quotient obtained thereby is the sine of the azimuth displacement from the meridian line.

For example, for the town of Ghazna, we transformed the ordinary sine and the versed sine of the longitudinal difference and obtained the amounts stated heretofore. Then we multiplied the transformed versed sine by the sine of the latitude of Ghazna and obtained the product 207;14,46,13,20. We divided this product by the total sine and obtained the quotient 3;27,15. Now, the sum of the latitude of Mecca and the colatitude of Ghazna is 78;5°, and the versed sine of this sum is 47;36,39; we added this (MS 311) to the quotient and obtained 51;3,54, which is the measure of the gauge. Since this gauge is less than the total sine, the azimuth of the **qibla** is south of the east-west line. Further, we multiplied 8;56,6, which is the difference between the total sine and the gauge, by itself and obtained the product 79;50,3, 12,36; then we added to this the square of the sine of the modified longitude and obtained the sum 737;8,38,49,25, whose (square) root is 27;8,41. We divided the product of the sine of the modified longitude times the total sine by this root and obtained the quotient 56;40,11. The arc (sine) of this quotient, which is 70; 49,16°, is the displacement of the azimuth line of the **qibla** from the true south westwards.

Proof of this method: Again, we let the semicircle ABG (Fig. 67) represent the western horizon of Ghazna, and let us imagine that AKG represents a semicircle of its meridian. We mark off arc AK equal

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to the colatitude of Ghazna, and arc KH equal to the latitude of Mecca. We join KE and draw HT parallel to it; then we draw HY perpendicular to EK. It is obvious that KE is the line of intersection between the plane of the meridian of Ghazna and the plane of the celestial equator, and that HT is the line of intersection of the plane of the meridian of Ghazna and the plane of the parallel of Mecca, and that HY is the sine of the latitude of

Mecca, and EY is the cosine of its latitude. Further, we mark
6 off arc FK equal to the longitudinal difference, and we join FE.
With E as center and with radius (MS 312) EY we draw the arc
YN; then we drop NO perpendicular to KE, and we produce
NO to meet the line TH at M. It is known that arc YN lies on a
9 circle which is equal to the parallel of Mecca, because it was
drawn with a radius equal to the cosine of its latitude, and since
YN is similar to arc FK, arc YN is a measure of the longitudinal
difference in the parallel (of Mecca). So NO is the sine of the
modified longitudinal difference, measured in that circle, and YO
is the versed sine of the longitudinal difference, which is also
12 measured in that circle. Hence YO is the transformed amount,
and HM is equal to it and is really in an analogous position in the
meridian circle of Ghazna.

We drop both HD and ML perpendicular to AEG. Then
15 HD is the sine of the sum of AK, the colatitude of Ghazna, and
KH, the latitude of Mecca. Therefore AD is the versed sine of
this sum. Further, we draw MC parallel to AG: then the triangle
HMC is similar to the triangle HDT, the triangle of daylight.
18 So the ratio of HM, the transformed versed sine, to MC is as
the ratio of the sine of angle HCM, the right angle, to the sine of
angle HMC, the colatitude of Ghazna. Hence MC is known.
Now, DL is equal to MC, and AL, the gauge, is the sum of AD
and DL. (It is called the gauge) because point L is

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on the line through the foot of the vertical from Mecca parallel
to the east-west line. Whenever it falls between the two points
A and E, (MS 313) the line that issues from E to the point which
3 is supposed to fall on it, ends up on the southern quadrant AB,
but if it falls beyond E, towards G, then that line ends up on the
northern quadrant BG.

Moreover, it is known that the segment between L and the
6 foot of the vertical from Mecca is equal to the sine of the modified
longitude, I mean NO. So if we separate LZ, which lies on the
prolongation of ML - though it really makes with it a right
angle, but if the semicircle AKG is revolved about the axis AEG
until it coincides with the eastern half of the horizon, then ML

9 would coincide with the said line and LZ would fall on the pro-
longation of ML - and if we join EZ and produce it to S, then
ES will be the line of the **qibla.** Now ZE is the hypotenuse of a
right triangle with legs ZL and LE; therefore ZE is known. But
the ratio of ZE to ZL is as the ratio of the sine of angle ZLE, the
12 right angle, to the sine of angle LEZ whose magnitude is that of
arc AS, the displacement of the azimuth line from the meridian
line. Therefore the azimuth is known from this proportion, and
that is what we wanted to prove.

Otherwise, if we like, we divide the product of the sine of
15 the modified longitude times the total sine by the difference
between the gauge and the total sine. The quotient obtained
thereby is a measure of the tangent of the displacement of the
azimuth from the meridian line.

For example, in the last computations for the city of Ghazna,
18 (MS 314) the product of the sine of the modified longitude times
the total sine was 1538;17,0. We divided this product by the
difference between the gauge and the total sine, which is 8;5[6,6],
and obtained the quotient 172;9,50, which is the tangent of the
displacement of the direction of the **qibla** at Ghazna from the
south. The arc (tangent) of this displacement is 70;47,9°.

21 Proof: We draw AW, a tangent line to the circle at A, and
we produce ES until it meets the tangent line at W. Then AW is
the tangent of arc AS. Further, the ratio of EL, the difference
between the gauge and the total sine, to LZ, the sine of

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the modified longitude, is as the ratio of EA, the total sine, to
AW, the tangent. Hence it (the tangent) is known.

If we wish to find the cotangent function, we multiply the
3 difference between the gauge and the total sine, and we divide the
result by the sine of the modified longitude, and what results is
the cotangent for the distance of the azimuth from the south
point.

An example of it for the preceding operation for Ghazna
(is) that we multiply the difference between the gauge and the
6 total sine by the total sine, and obtain the product 536;6,0, then
we divide the product by the sine of the modified longitude and

obtain the quotient 20;54,37, which is the cotangent of the displacement of the azimuth (of the **qibla**) at Ghazna from the south point. The arc of this azimuth is 70;47,11°.

Its proof is evident, because if we draw at point B a tangent line (MS 315) to the circle, then the segment of this line between point B and the line ESW is the cotangent of arc AS. That segment is the line BQ (Fig. 67), and its ratio to BE, the total sine is as the ratio of LE to ZL. It is the practice of calculators to express the cotangent in digits and not in (sexagesimal) parts, and the ratio of the digits to the parts is one fifth. So if we take one fifth of the parts of the cotangent, by multiplying those parts by twelve minutes, then the digits in the last example would be 4;10,55.

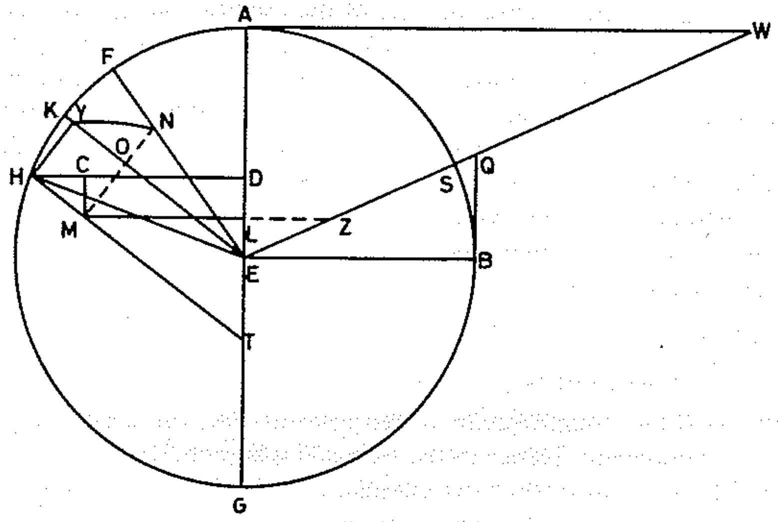

Figure 67

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The practical method for (MS 316) the determination of the azimuth of the **qibla** is evidently as follows: If AHG is the

meridian line in a circle whose plane is parallel to the horizon, and if we take arc AK equal to the colatitude of our town, arc KH equal to the latitude of Mecca, arc KF equal to the longitudinal difference between them, and if we join the segments FE and KE, and if we construct HT parallel to KE, and HY perpendicular to KE, and if, with center E and at a distance EY, we draw the arc YN, and if we construct NO perpendicular to KE and extend NO to meet TH at M, and further, if we construct MLZ perpendicular to AG, and we make LZ equal to NO, and if we produce EZ to meet the circumference of the circle at S, then ES will be the line of prayer.

Let us reproduce the first figure (Fig. 65) which was given to illustrate the method described in the zījes for the determination of the azimuth of the **qibla,** and let us draw in it the great circle arc BMK (Fig. 68). Then the ratio of the sine of TM, the complement of the latitude of Mecca, to the sine of MK is as the ratio of the sine of the quadrant TL, to the sine of LH, the longitudinal difference. Therefore arc MK, the modified longitude, is known. Also, the ratio of the sine of BM, the complement of the modified longitude, to the sine of ML, the latitude of Mecca, is as the ratio of the sine of the quadrant BK to the sine of KH, where KH is called the modified latitude. So arc KH is known. Hence KE, which is equal to the difference between KH (MS 317) and HE, the latitude of the town, is known, and its complement KA is known. Further, the ratio of the sine of BM, the cosine of the modified longitude, to the sine of MS, the altitude of Mecca, is as the ratio of the sine of the quadrant BK to the sine of KA, the complement of that difference. Hence MS is known, and its complement ME is the distance between the town and Mecca. Also, the ratio of the sine of ME to the

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sine of MK, the modified longitude, is as the ratio of the sine of the quadrant ES to the sine of SA, the displacement of the azimuth of the **qibla** from the meridian line.

We repeat the example. The sine of the modified longitude, which was obtained above, is 25;38,17. Its arc sine is 25;38,17. Its arc sine is 25;17,47°, and its complement is 64;42,13° and

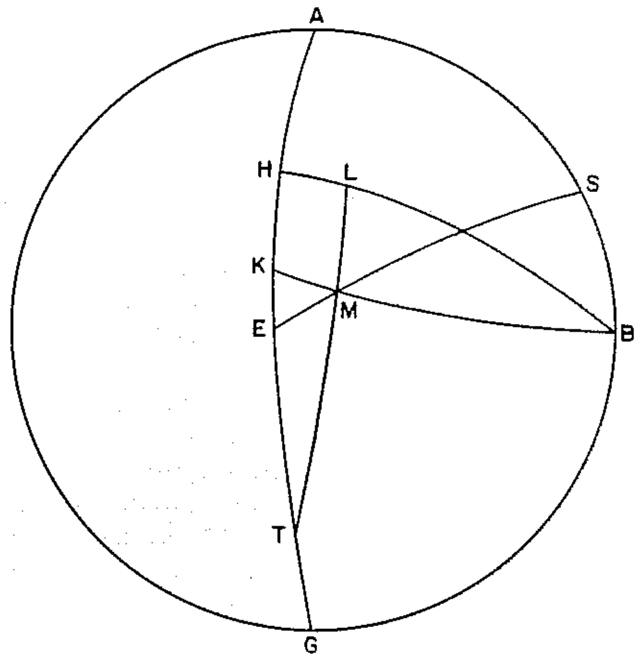

Figure 68

the sine of this is 54;14,48. We multiplied the sine of the latitude of Mecca by the total sine and obtained the product 1329;8,0; then we divided this product by the cosine of (MS 318) the modified longitude, and obtained the quotient 24;30,6. The arc (sine) of this quotient, 24;6,7°, is the modified latitude, and the difference between this and the latitude of Ghazna is 9;28,53°. The complement of this difference is 80;31,7°; we multiplied its sine, which is 59;10,49, by the cosine of the modified longitude and obtained the product 3210;19,58,5,12, then we divided this product by the total sine and obtained the quotient 53;30,19, whose arc (sine) is 63;5,40°. The complement of this arc is 26;54,20°; it is the direct (great circle) distance between Ghazna and Mecca, which is equivalent to 1524;38,53 miles or 508;12,58 farsakhs. We divided the product of the sine of the modified longitude times the total sine by 27;9,4, the sine

of the distance, and obtained the quotient 56;39,23.

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The arc (sine) of this quotient is 70;46,56°; it is the measure of the displacement of the azimuth (line) of the **qibla** from the meridian line.

(Method for Architects): These methods are sufficient for those who wish to use elaborate methods, but as architects and artisans can not work out the precise amounts which we have derived, they may proceed as follows: Let them draw on a polished stable surface a circle, and place in it the diameter which represents the meridian line (Fig.69), and let them divide the radius which joins the center to the south point into three equal parts. Then, at Ghazna, let them count one of those parts from the center, and let them erect at its extremity a perpendicular

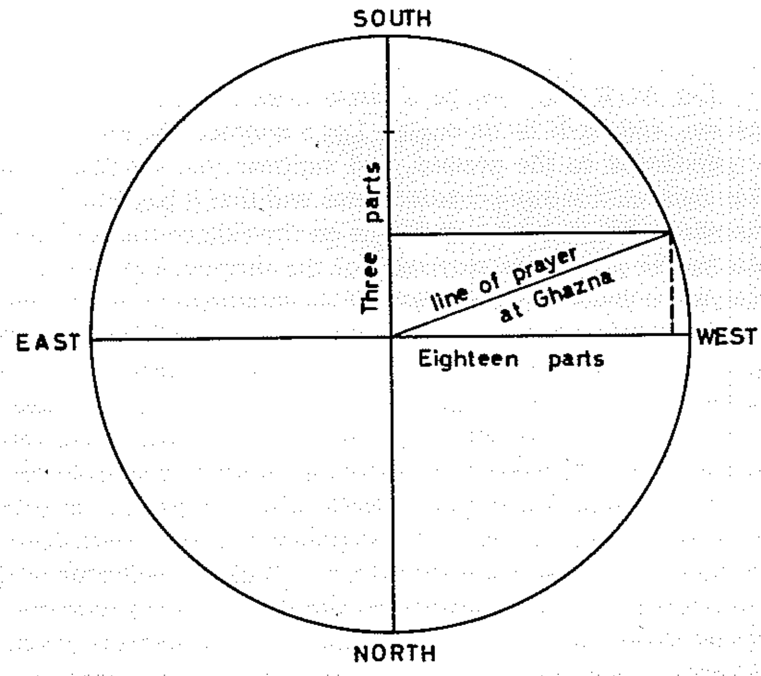

Figure 69

directed westwards and extended to meet the circumference. Let them join its point (MS 319) of intersection with the circumference to the center by a straight line. Then this line gives the direction of prayer, and the base of the **miḥrāb** (prayer niche) wall is perpendicular to it. For better precision, let them divide the radius which joins the center to the west point into eighteen equal parts, and let them count one part from the west point and erect at its extremity a perpendicular to the diameter, directed southwards. The intersection of this perpendicular with the circumference gives the azimuth point with better precision. Then let them draw the lines described earlier in this paragraph. Here follows the figure.

(Determination of the local meridian): If they need the meridian line, the method of its determination by means of

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the Indian Circle (MS 320) is in current use by them, but if they want a method with one measurement of time instead of two time measurements, the following method leads to it:

Let AGB (Fig. 70) be a circle drawn in the plane of the horizon, and let E be its center. Let a gnomon be erected on it perpendicular to the ground, and suppose that its shadow has been measured at any proposed time, and that it has fallen on the diameter AEB, where point A falls on the side of the sun, and B falls on the side of the extremity of the shadow. Let us suppose that AG is equal to the altitude of the sun at the time of observation, and that AO is equal to the colatitude of our town. We draw the diameter OEF, and we mark off each of the arcs OD, and FZ equal to the sun's declination. If the declination is northerly, the arcs are directed towards B, but if it is southerly, then they are directed towards A. Further, we join D to Z, and we draw GH parallel to AB. Then we drop HT, and GK perpendicular to AB; and with center E and at a distance ET, we draw the semicircle TLM. On KE as diameter, we draw the semicircle KLE. Both semicircles are drawn on the side of AB corresponding to the particular half of the daylight then in progress (i.e. forenoon or afternoon), and the two circles will intersect. Let L be the point of intersection, and let us draw

the straight line ELC, then this line is a segment of the meridian line. (MS 321)

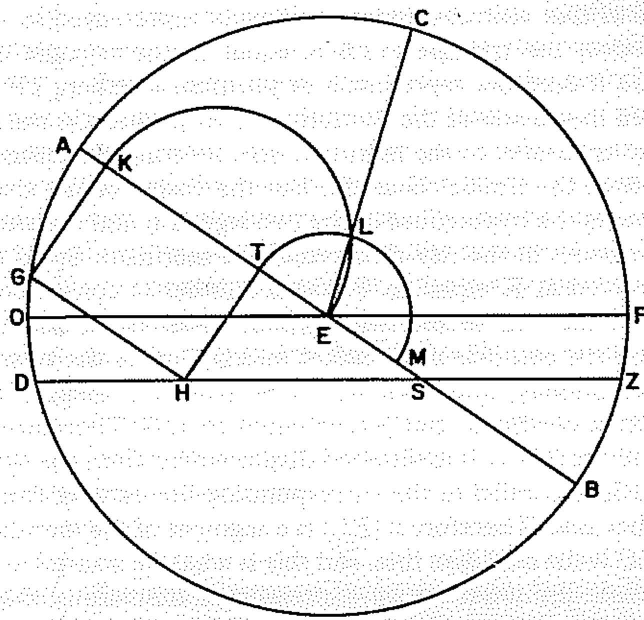

Figure 70

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Having discussed and explained the properties of the triangle of daylight and those of the triangle of time, it is easy to give a proof of the validity of this construction. If the semicircle AGB is taken to represent the semicircle of the meridian, then OE represents the line of intersection of the plane of the meridian and that of the celestial equator, and DZ represents the line of intersection between its plane and that of the sun's daily path (or day-circle). Hence DS is the «diameter» of the daylight triangle in this day-circle; angle S is equal to the colatitude of the town, and ES is equal to (MS 322) the sine of the rising amplitude. Though these have not been represented in their true positions, their magnitudes have been accurately determined in other positions. Because AG was constructed equal to

9 the altitude of the sun at the time of observation, GK and HT
are both equal to the sine of the altitude, KE is equal to the
cosine of that altitude, and each is truly represented in its posi-
tion. Also, the triangle HTS is equal to the triangle of time,
thought it does not represent it in position. Further, TE is what
is called the «share» of the azimuth, and its position in the triangle
12 of time is parallel to the meridian line, meeting the cosine of the
altitude at the time at point K. But the cosine of the altitude at
the time is the hypotenuse of the two legs, TE and the sine of the
displacement in the day-circle from the meridian. So, if a line is
constructed at K equal to TE, and another is constructed at E
15 equal to the sine of the displacement from the meridian, they
meet on the east side of G before midday, or on the west side of
G after midday, and both would be truly represented in their
respective positions. But EL is equal to ET. Therefore TK is
equal to the (above-) mentioned displacement from the meridian.
18 Also, EL is parallel to the corresponding line issuing from K on
the other side. Therefore it (EL) is a segment of the meridian line.
So ELC is the meridian line, and this is what we wanted to clarify.

Thus the work we have presented, concerning the verifi-
cations of the longitudes of towns and their latitudes,

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is beneficial to the majority of Muslims because it helps them to
determine the direction of the **qibla** accurately and to hold their
prayers accordingly, free from the blemish of a misconducted
investigation. It is also of special significance to the people of
3 Ghazna, because we set out to determine its position correctly.
Further, apart from its benefit to Muslims, it is useful to the
People of the Book as well as others. For example, the direction
of Jerusalem is as important to the Jews as the direction of the
Kaᶜba is for us, and if its longitude and latitude are corrected,
the direction of the **qibla** in the Jewish synagogues can be
accurately determined. Further, the east-west line is as important
6 for the Christians as the direction of the **qibla** is for us, because
their churches face east, and the meridian line (is of special
significance) to the Ḥarrānians, known as the Sabians. Therefore,
it is beneficial to most people professing the main and most

rewarding religions, and I have no doubt that it is useful also
9 to people of all faiths.

When one determines accurately the longitude and the
latitude of one's town, one can compute the noontime and the
times of the midafternoon, the setting of the twilight, and the
rise of the dawn which ushers in the beginning of the fast after
the morning prayer. Further, one can determine the times for
12 the visibility of the new moons, though the Islamic law com-
mands their determination with the naked eye and not by
computation, because the Prophet, the prayers of God upon him,
said: «We are people who neither write nor compute». Hence, the
month is so, and so, and so, showing out his ten fingers thrice;
15 then so, and so, and so, but folding his thumb in the third (show
of his hands).

If we pass on from a religious benefit to a worldly one,
then we refer to what we mentioned before concerning the
determination of the proper

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direction for travel to specified destinations, in search of material
prosperity or to ward off an iminent danger. Further, it is re-
quired by the professional astrologers for locating the positions
of the planets, and for finding the correct center for the cardines,
3 and so on. Also, the astrologers need it for time determination,
such as the times of nativities, and those of (year) - transfers,
conjunctions, oppositions, intermediate quadratures, semi-
quadratures, etc. As the profession of astrology is based on weak
6 foundations, as its subsidiary branches are defective, as the mea-
surements made are inaccurate, as the predictions made are based
on probability and not on certainty, as its object is the study of
the figures for the relative distributions of the planets, and as the
position of each planet depends on its proper motion and on its
position relative to the horizon, so their predictions can never be
valid unless the data of their object are accurate. How can such
9 an object, valid at an anonymous place for which it was compu-
ted, be valid for the horoscopes of conjunctions and oppositions
at another place where these are, indeed, different from those for
which it was computed! If horoscopes are valid at another place,

then the object of astrology would be the computation of the astrologers; it would not be the figures for the positions of the 12 planets, and if this attitude is carried too far, astrology would be regarded as identical with the practice of augury by the **Hashtmarj** lines, or by the rousing of a flock of birds for predicting good or evil omens.

Our work is also indispensable to professional observers for investigating the validity of computations made by calculators 15 that follow the tradition of the Sindhind. Indeed, these calculators will be put to shame when their computations and those of others are put to the test by actual observation of the eclipses of the two luminaries, because you will then see the discrepancies between the actual times of lunar eclipses and the times that they had computed for them, and there will be discrepancies, too, in 18 the times and magnitudes of solar eclipses. They overlook such matters because they are of a difficult and serious nature. But it is catastrophic to them if an eclipse happens to take place near the horizon, for then they will be stunned with the falsity of their calculations, and they will not be able to find an excuse for their error, or the reason for their wrong prediction.

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For example, those Khurāsānian calculators are ignorant of the process of transformation from certain towns to others, because they have preferred material gain to scientific knowledge. They have been contented with imitating the work of others 3 without doing any research of their own. Their computations follow the rules given in al-Battānī's zīj, which is based on Raqqa, whose longitude, as given in the books, is seventy-three degrees. As mentioned previously, the longitude of Baghdad is between seventy and eighty degrees. They have taken the distances of their towns from Raqqa to be less than their distances from 6 Baghdad by three degrees, but they should have taken the former distances to be more than the latter by seven degrees. So they have made a total error of ten degrees, which is the sum of the error in excess and that in defect. This total error, measured in moments of time, amounts to two thirds of an hour. That is why they had said, concerning a lunar eclipse that occurred in Jumada

I, year four hundred ten (of the Hijra), that it would start at Ghazna after a lapse of seven hours and a half after nightfall. However, when I observed it, the altitude of Capella above the eastern horizon was slightly less than 66° when the cut at the 12 edge of the full moon had become visible; the altitude of Sirius was 17°, that of Procyon was 22°, and that of Aldabaran was 63°, where all altitudes were measured from the eastern horizon. All these observations show that the eclipse must have started after a lapse of approximately eight hours. Also, they said that its final 15 clearance would be after a lapse of ten hours and a quarter. However, the night hours then were almost equal to the day hours because the sun at that time was in its final stages through the constellation of Virgo. According to their claim, the final clearance should have occurred before the end of that night by one hour and one half and one quarter, but it was apparent to 18 the eye that the world had lightened up, the stars had disappeared, the sun was about to rise, and the moon, with a trace of the eclipse over its surface, was about to set behind the mountains which ultimately screened it from my sight, and I was unable to observe the exact time of its final clearance.

21 Again, though they had not discussed the solar eclipse that took place in Dhū al-Qaᶜda, year four hundred nine of the Hijra, the reserved amongst them said that it would occur below the horizon of Ghazna, and that it would not be seen there. However, it happened that we

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were near Lamghān, between Qandahār and Kābul, in a valley surrounded by mountains, where the sun could not be seen unless it was at an appreciable altitude above the horizon. At 3 sunrise, we saw that approximately one third of the sun was eclipsed and that the eclipse was waning. The main source of error is their ignorance of the position of Raqqa relative to Baghdad, and their ignorance, too, of the mode of development of a solar eclipse. They are incapable of the precision required for it, and they are too small (-minded) to appreciate its majesty and beauty.

6 This is why Galen wrote a book in which he stated that a

learned physician must also be a philosopher, that is, a man
seeking wisdom and fond of it. To them, philosophy, I mean
wisdom, is the knowledge of existing objects by the perception of
the universal truths governing their existence. If one makes a
9 thorough inquiry, one permits oneself to say that everyone who is
interested in one of the sciences must also be a philosopher who
has studied the first principles of each of the sciences, though
one's span of life is insufficient for a detailed study of the bran-
ches of all the sciences.

12 If the people mentioned above had studied the science of
history and chronology and if they had known of the existence of
different kingdoms and the roads that lead to them, they would
have realized from their studies that the road from Baghdad to the
capitals and the towns of Syria as well as the roads to the (land
of the) Byzantines go through Raqqa, and that the caliphs during
15 their conquests have made it one of their resting stations, and
that the (land of the) Byzantines is farther from Khurāsān than
Baghdad, and that stations on the road from Baghdad to the
Byzantines are, similarly, farther from Khurāsān. But, when I
asked one of them where would Raqqa be and to which country
18 does it belong, I found that he had acquired only half the truth
about it, and that the full truth could not be gleaned from his
scanty knowledge. Yet he refers to it when he uses al-Battānī's zīj
for calculating distances between towns. Also, I found that Raqqa
had meant to him no more than the Cupola to the fanatics of the
Sindhind, a mere name for an unsubstantial thing, and that his
21 points of view did not conform to the rules of astronomy and were
incompatible with the laws of Nature. Praise and glory be to Him
who has not declined his favor to a man who was more astray
than a beast.

As we have corrected longitudinally and latitudinally the
displacement of Ghazna from Baghdad, in order to find the
correct direction of the **qibla** at Ghazna,

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since the displacement of Mecca from Baghdad was known, we
must also correct the displacement of Ghazna from those places
upon which zījes are based, so that an observer of the stars

3 there will not use the wrong data for locating them.

We say: As to the computations of the Sindhind, they are
based on the Cupola, which is regarded by the calculators as the
middle of the inhabited world. They have all agreed that it is
6 situated east of Baghdad by twenty units of time, that is, by one
hour and one third. Hence, Ghazna would be east of the Cupola
by four units of time and one fifth and one sixth of one unit of
time, that is, by one quarter and one third of one tenth of an hour.

As to the computations of the Westerners, they are derived
9 from the book Almagest and from Theon's Canon, which are
based on Alexandria, in Egypt. In the fifth chapter of the Alma-
gest, Ptolemy considered its latitude to be 30;58°; also, having
made use of Babylonian observations, he considered the longi-
tudinal difference between it and Babylon to be one half and one
12 third of an hour, which is equivalent to twelve units of time and
one half. The moderns have used for this distance thirteen units
of time and three quarters of a unit, that is, one half and one
quarter and one sixth of an equinoctial hour. It would have been
preferable if they had found the correct longitudinal difference
between Alexandria and Shammāsiya, in the vicinity of Baghdad,
15 but this is not known to us because they did not report it. If they
have adopted this increase because of the distance between
Babylon and Baghdad, they overestimated it, because Babylon
is not far from Baghdad. However, I think that this distance is
more than that used by Ptolemy.

18 As to (the computations based on) Raqqa, there is some
confusion in al-Battānī's zīj, and his data are different from

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those given above. For, in his tables on the longitudes of cities,
he recorded the following longitudes: 60;30° for Alexandria, 73°
3 for Raqqa, 79° for Babylon, and 80° for Baghdad. It follows from
this that the longitudinal difference between Alexandria and
Baghdad is 19;30°, that between it and Babylon is 18;30°, and
that between it and Raqqa is 12;30°. But, when he used this for
the derivation of the solar motion, he considered it to be 10;0°,
6 because he claimed that midday at Raqqa is ahead of midday at
Alexandria by two thirds of an hour.

XXIV. DETERMINATION OF THE LONGITUDINAL DIFFERENCE BETWEEN BAGHDAD AND RAQQA

If we follow for this purpose the method we used in similar determinations mentioned above, we find that the latitudinal
9 difference between Baghdad and Raqqa is 2;36°, its chord is 2;43,21, and the square of this is 7;24,43,18,21. The distance between Baghdad and Raqqa is 130 farsakhs, because the distance from Baghdad to Anbār is 12, then 19 to Hīt, then 27 to ᶜĀna,
12 then 39 to Raḥba, and then 23 to Raqqa. If we deduct twenty farsakhs from their sum, approximately one sixth of it, the remainder will be 110 farsakhs, which is equivalent to 330 miles or 5;49,34°. The chord of this is 6;5,54, and its square is 37;11,
15 22,48,36. Hence the difference between the two squares is 29;46, 39,35,15. We multiplied this difference by the cosine of the latitude of Baghdad and obtained the product 1491;17,54,27,9, 33,0; then we divided this by the cosine of the latitude of Raqqa and obtained the quotient 30;43,43,59,26, whose root is 5;32,36.
18 We multiplied this by the total sine and obtained the product 332;36,0; then we divided this product by the cosine of the latitude of Baghdad and obtained the quotient 6;38,28. This is the chord of an arc of 6;20,43°, which is the longitudinal difference between Baghdad and Raqqa. This is not far from that
21 recorded in the zīj (of al-Battānī) because if we adopt 80° for the longitude of Baghdad, the longitude of Raqqa, according to our derivation, would be 73;39,17°. Therefore 73° is a reliable longitude for Raqqa, and it is corroborated by al-Hāshimī's report which we mentioned above (in 203;10).

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XXV. DETERMINATION OF THE LONGITUDINAL DIFFERENCE BETWEEN RAQQA AND ALEXANDRIA

3 As to the determination of the longitudinal difference between Raqqa and Alexandria, it is known that the latitudinal difference between them is 5;3°; its chord is 5;17,12, and the square of this is 27;56,55,50,24. The distance between them is approximately 750 miles; it is made by way of Ḥims, Dimashq (Damascus), Ṭabariya (Tiberias), Ramla, and Miṣr (Cairo),
6 though the road is not a straight stretch. It is 154 from Raqqa to Ḥims, then 86 to Damascus, then 66 to Tiberias, then 67 to Ramla, then 297 to Fusṭāṭ Miṣr, and then 80 to Alexandria. If we
9 deduct one sixth of the sum of these miles, there remains 628 which is equivalent to 11;4,56°. The chord of this is 11;31,4, and its square is 132;39,33,8,16. Hence the difference between the two squares is 104;42,37,17,56. We multiplied this difference
12 by 51;26,53, the cosine of the latitude of Alexandria, and obtained the product 5387;8,39,59,53,18,56; then we divided this product by the cosine of the latitude of Raqqa and obtained the quotient 111;0,16,27,49 whose root is 10;32,9. We multiplied this root by the total sine and obtained the product 632;9,0; then
15 we divided this product by the cosine of the latitude of Alexandria and obtained the quotient 12;17,14. This is the chord of an arc of 11;45,15°, which is the longitudinal difference between Alexandria and Raqqa.

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That difference leads approximately to what is given in al-Battānī's zīj, because if we add this amount, derived by approximation, to the longitude recorded in it for Alexandria,
3 then the total is 72;15,15°, which is approximately equal to the longitude of Raqqa, and if we deduct it from the longitude of Raqqa, there remains 61;14,45°, which is approximately equal to the longitude of Alexandria.

The heart has gained confidence in the records in al-Battānī's zīj, and I am now more inclined to think that the longitudinal difference between Alexandria and Baghdad is more than that used by the observers at Shammāsiya.

Thus the following rule has been established: If we wish to calculate the corresponding time at Ghazna, we must discount from the date at Alexandria the amount of 43;52 units of time or 2;55,30 hours; from the date at Raqqa, we discount 31;22 units of time or 2;5,30 hours; from the date at Baghdad, we discount 24;22 units of time or 1;37,30 hours; from the date at the Cupola, we discount 4;22 units of time or 0;17,30 hours. Similarly, we obtain corresponding dates for the dates at all other towns, if their longitudes and their latitudes have been verified.

There is no harm in illustrating that by an example, say the time of an historic event which requires astronomical measurements, though human limitation restricts the ambition for a perfect achievement. This is the time of the sun's transit through the point of the autumnal equinox, in the constellation of Libra. I shall relate what is known to me about observations of it, though these were made at different times, and some of them were lacking in accuracy. I shall defer my criticism of these and will discuss their relative merits in another place, more appropriate for it than this book.

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XXVI. (OBSERVATIONS OF THE AUTUMNAL EQUINOX)

Observations of Hipparchus at Rhodes: The first of his observations of this equinox, as reported by Ptolemy in the third treatise of the Almagest, was held in the island of Rhodes, and he also mentioned in the fifth treatise that it falls on the meridian of Alexandria. Its date is the sunset of Tuesday, the last day of Mesore, the twelfth of the Coptic months, in the year five hundred eighty-six of Buchtanaṣṣar (Nabonassar). Since the longitudinal difference between Ghazna and Alexandria, expressed in day minutes, which are called **jahri,** is 7;18,44, the date of this equinox at Ghazna is the same Tuesday at 22;18,44 (day minutes) post meridian. The date of the second observation is sunrise on Saturday, the first of the epagomenal days, in the year five hundred eighty-nine of Buchtanaṣṣar. The corresponding date at Ghazna is Friday, the last day of Mesore, at 52;18,44 (day minutes) post meridian.

The date of the third observation is noontide of Sunday, the first of the epagomenal days, in the year five hundred ninety of Buchtanaṣṣar. The corresponding date at Ghazna is Sunday at 7;18,44 (day minutes) post meridian.

The date of the fourth observation is the middle of the night that preceded the morning of Sunday, the fourth of the epagomenal days, in the year six hundred one of Buchtanaṣṣar. The corresponding date at Ghazna is Saturday, the third of the epagomenal days, at 37;18,44 post meridian. Ptolemy mentioned that this (observation) was conducted very thoroughly.

The date of the fifth observation is sunrise on Monday, the fourth

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of the epagomenal days, in the year six hundred and two of Buchtanaṣṣar. The corresponding date at Ghazna is Sunday, the third of the epagomenal days, at 52;18,44 post meridian.

The date of the sixth observation is sunset of Thursday, the fourth of the epagomenal days, in the year six hundred and five of Buchtanaṣṣar. The corresponding date at Ghazna is Thursday at 22;18,44 post meridian.

Observations of Ptolemy at Alexandria: The date of the first of his two observations is eight hours after the beginning of Wednesday, the seventh of Athyr, the third of the Coptic months, in the year eight hundred eighty of Buchtanaṣṣar. The corresponding date at Ghazna is Wednesday at 12;18,44 (day minutes) post meridian.

The date of the second observation is one hour after the beginning of Friday, the ninth of Athyr, in the year eight hundred eighty-seven of Buchtanaṣṣar. The corresponding date at Ghazna is Thursday, the eighth of Athyr, at 54;48,44 (day minutes) post meridian.

Observations at Shammāsiya and Baghdad: Yaḥya bin Abi Manṣūr found it (after) midday of Sunday, the twenty-fifth of Pharmuthi, the eighth of the Coptic months, in the year one thousand five hundred seventy-seven of Buchtanaṣṣar

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by four fifths of an hour. The corresponding date at Ghazna is Sunday at 6;3,44 (day minutes) post meridian.

The date of the second, made by an anonymous observer, is one hour before midday of Monday, the twenty-fifth of Pharmuthi, in the year one thousand five hundred seventy-eight of Buchtanaṣṣar. The corresponding date at Ghazna is Monday, at 1;33,44 (day minutes) post meridian.

The date of the third, reported in the book **Sanat al-Shams** (The Solar Year), is one hour after sunset of Tuesday, the twenty-fifth of Pharmuthi, in the year one thousand five hundred seventy-nine of Buchtanaṣṣar. The corresponding date at Ghazna is Tuesday, at 21;33,44 post meridian.

Observation of Khālid at Damascus: Khālid bin ᶜAbd al-Malik al-Marwarūdhī found it on Thursday, the twenty-sixth of Pharmuthi, in the year one thousand five hundred eighty of Buchtanaṣṣar at twelve hours and four fifths of an hour before noontime. The longitudinal difference used in practice between

Damascus and Baghdad is ten degrees, and the position of the former relative to Raqqa and Alexandria is not inconsistent with that (difference). Hence the corresponding date of this equinox at Ghazna is Wednesday, the twenty-fifth of Pharmuthi, at 33;43,44 (day minutes) post meridian.

An Anonymous Observation at Baghdad: The date at it was found to be at three hours and one fifth and one sixth from the nightfall of Thursday,

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the twenty-ninth of Pharmuthi, in the year one thousand five hundred ninety-one of Buchtanaṣṣar. The corresponding date at Ghazna is Wednesday, the twenty eight of Pharmuthi at 27;28,44 (day minutes) post meridian.

Observation of Muḥammad bin ᶜAlī at Nīshāpūr: Muḥammad bin ᶜAlī al-Makkī found it (the date of the autumnal equinox) at it (Nīshāpūr) to be at noontide of Sunday, the last day of Pharmuthi, in the year one thousand five hundred ninety-nine of Buchtanaṣṣar. As the longitude of Nīshāpūr was settled, the corresponding date at Ghazna is Saturday, at 1;33,44 post meridian.

Observation of Banū Mūsā at Surra-man-ra'ā: They found it at Surra-man-ra'ā to be at noontide of the second Tuesday of the month of Pachons, the ninth of the Coptic months, in the year one thousand six hundred seven of Buchtanaṣṣar. As Surra-man-ra'ā is west of Baghdad by one quarter of a degree, the corresponding date for this equinox at Ghazna is Tuesday at 13;6,14 post meridian.

Observation of al-Battānī at Raqqa: He found it at seven hours and a quarter of an hour from the beginning of the night of Wednesday, the eighth of Pachons, in the year one thousand six hundred thirty of Buchtanaṣṣar. The corresponding date at Ghazna is Tuesday, the seventh of Pachons, at 38;21,14 (day minutes) post meridian.

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Observation of Sulaimān bin ᶜIṣmat at Balkh: He found it, at Balkh, at seven hours and three fifths of an hour from the

3 beginning of Wednesday, the ninth of Pachons in the year one thousand six hundred thirty-six of Buchtanaṣṣar. The corresponding date at Ghazna is Wednesday, at 3;43,14 post meridian.
6 Observation of Abū al-Ḫusain (sic) al-Ṣūfī at Shīrāz: In the first observation he found it at five hours from the beginning of Sunday, the twenty-ninth of Pachons, in the year one thousand seven hundred eighteen of Buchtanaṣṣar. So, according to our decision concerning the longitude of Shīrāz, the corresponding date at Ghazna is Sunday, at 5;8,8,40 post meridian.
9 In the second observation, he found it at sunset of Monday, the twenty-ninth of Pachons, in the year one thousand seven hundred nineteen of Buchtanaṣṣar. The corresponding date at Ghazna is Monday, at 17;38,8,40 post meridian.
12 Observation of Abū al-Wafā' at Baghdad: He found it at Baghdad, at three hours from the beginning of Friday, the last day of Pachons in the year one thousand seven hundred twenty-two of Buchtanaṣṣar. The corresponding date at Ghazna
15 is Thursday, the twenty-ninth of Pachons, at 56;33,44 (day minutes) post meridian.

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Observation of Abū Raiḥān at Jurjāniya: I found it at seven hours from the beginning of Monday, the tenth of Payni, the
3 tenth of the Coptic months, in the year one thousand seven hundred sixty-four of Buchtanaṣṣar. The corresponding date at Ghazna is Monday, at 4;13,35 (day minutes) post meridian.
6 Observation of Abū Raiḥān at Ghazna: I found it, at Ghazna, after midday of Thursday, the tenth of Payni, in the year one thousand seven hundred sixty-seven of Buchtanaṣṣar, by 47;30 (day minutes), which is equivalent to 19;0 hours, or 285 units of time (**azmān**).
9 May Almighty God, with His grace and ample munificence, help me in my researches for verifying the celestial motions by allowing me to continue my observations. It is He whose bounty and reward are sought, and whose severe retribution awes; it is
12 He who is requested for guidance to deserve His blessings and to avoid His indignation.

The book **taḥdīd nihāyāt al-amākin li-taṣḥīḥ masāfāt al-masākin** has been completed, and I have concluded its copying at Ghazna, seven days before the end of Rajab, in the
15 year four hundred and sixteen (of the Hijra).

GENERAL INDEX

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